Optimizing Area: Four-Sided Enclosure

1. Complete the tables below for each scenario.

| Scenario A: A farmer has 32 metres of fencing to enclose an area. What dimensions would maximize the area? Complete the table below for possible dimensions of the four-sided enclosure. Use whole numbers only. |
|---|---|---|---|
| Width (m) | Length (m) | Perimeter (m) | Area (m²) |
| 1 | 15 | 32 | 15 |
| 32 | 32 | 32 | 32 |
| 32 | 32 | 32 | 32 |
| 32 | 32 | 32 | 32 |
| 32 | 32 | 32 | 32 |
| 32 | 32 | 32 | 32 |
| 32 | 32 | 32 | 32 |
| 32 | 32 | 32 | 32 |
| What dimensions maximize the area? |

| Scenario B: A farmer has an area of 144 m² that needs to be enclosed. What dimensions will minimize the perimeter? Complete the table below for possible dimensions of the four-sided enclosure. Use whole numbers only. |
|---|---|---|---|
| Width (m) | Length (m) | Perimeter (m) | Area (m²) |
| 1 | 144 | 290 | 144 |
| 144 | 144 | 144 | 144 |
| 144 | 144 | 144 | 144 |
| 144 | 144 | 144 | 144 |
| 144 | 144 | 144 | 144 |
| 144 | 144 | 144 | 144 |
| 144 | 144 | 144 | 144 |
| What dimensions minimize the perimeter? |

2. Conclusions: for a four-sided enclosure,
   - Given the perimeter, the shape of a ___________ will maximize area
   - Given the area, the shape of a ___________ will minimize perimeter

- Complete the table below with the calculations for a four-sided enclosure:

<table>
<thead>
<tr>
<th>number of sides</th>
<th>side length</th>
<th>perimeter</th>
<th>area</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>s</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Optimizing Area: Three-Sided Enclosure

3. Complete the tables below for each scenario. Use the following diagram to set up the table:

![Diagram of a three-sided enclosure]

**Scenario C:** A farmer has 16 metres of fencing to enclose an area. The area only needs to be enclosed on three sides because it is built against the wall of a barn. Complete the table below for possible dimensions of the three-sided enclosure. Use whole numbers only.

<table>
<thead>
<tr>
<th>Width (m)</th>
<th>Length (m)</th>
<th>Perimeter (m)</th>
<th>Area (m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
<td>16</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td></td>
<td></td>
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<td></td>
<td>16</td>
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<td></td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What dimensions produce the maximum area?

**Scenario D:** A farmer has 20 metres of fencing to enclose an area. The area only needs to be enclosed on three sides because it is built against the wall of a barn. Complete the table below for possible dimensions of the three-sided enclosure. Use whole numbers only.

<table>
<thead>
<tr>
<th>Width (m)</th>
<th>Length (m)</th>
<th>Perimeter (m)</th>
<th>Area (m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18</td>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>16</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>16</td>
<td></td>
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<td>16</td>
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<td></td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What dimensions produce the maximum area?

4. Conclusions:

- An optimal rectangle with a three-sided enclosure always forms a _________ where the longest side is _________ the _________ side.

- Complete the table below for an optimal rectangle enclosed on three sides:

<table>
<thead>
<tr>
<th>number of sides</th>
<th>shorter side length</th>
<th>longer side length</th>
<th>perimeter</th>
<th>area</th>
</tr>
</thead>
</table>
5. Complete the tables below for each scenario. (use linking cubes)

Scenario E: A rectangular prism has volume of 8 m³. Use whole numbers only to complete the table below for possible dimensions of the prism.

<table>
<thead>
<tr>
<th>V (m³)</th>
<th>Length (m)</th>
<th>Width (m)</th>
<th>Height (m)</th>
<th>Surface area (m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What dimensions produce the minimum surface area?

Note: Conversely, given the surface area of a prism, what shape will maximize the volume?

6. Conclusions:
- Given the surface area, the shape of a ________________ will maximize volume.
- Given the volume, the shape of a ________________ will minimize surface area.
- Complete the table below with the calculations for a four-sided enclosure:

<table>
<thead>
<tr>
<th>side length</th>
<th>area of one surface</th>
<th>surface area</th>
<th>volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Practice & Homework:
1. What is the maximum area of a four-sided enclosure using 48 metres of fencing?

2. The area of an optimal rectangle is 64 m². What is the perimeter if the area is enclosed on four sides?

3. The side length of a rectangle, enclosed on four sides, is 15 m. What is the minimum perimeter and what is its maximum area?

4. A rectangle is to be created such that it has an area of 100 cm².
   a) What dimensions would minimize the perimeter?
   b) What is the minimum perimeter?

5. The area of a three sided enclosure 8450 m².
   a) What dimensions produce the maximum area of the enclosure?
b) What is the perimeter?

6. The longest side of an optimal rectangle, enclosed on three sides, is 14 m.
   a) What is the perimeter?
   b) What is its maximum area?

7. What is the minimum surface area of a rectangular prism with a volume of 729 cm³?

8. The surface area of an optimal prism is 150 cm². What are the dimensions of the prism?

9. The surface area of a cube is 2646 cm². Determine its maximum volume.

10. The volume of an optimal rectangular prism is 13824 cm³. What are the dimensions of the prism?

11. The maximum volume of a rectangular prism is 1331 cm³. Determine its minimum surface area.

12. Complete the table below for a four-sided enclosure with maximum area.

<table>
<thead>
<tr>
<th>Perimeter</th>
<th>Length</th>
<th>Width</th>
<th>area</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 64 m</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) 500 km</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) 88 cm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) 512 cm</td>
<td></td>
<td></td>
<td>625 cm²</td>
</tr>
<tr>
<td>e)</td>
<td></td>
<td></td>
<td>72 cm²</td>
</tr>
<tr>
<td>f)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

13. Complete the table below for a three-sided enclosure with maximum area.

<table>
<thead>
<tr>
<th>Perimeter</th>
<th>length</th>
<th>Width</th>
<th>area</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 64 m</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) 72 cm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) 100 km</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d)</td>
<td></td>
<td></td>
<td>200 m²</td>
</tr>
<tr>
<td>e)</td>
<td></td>
<td></td>
<td>450 m²</td>
</tr>
</tbody>
</table>

Extended Thinking

14. What dimensions for a cylinder would maximize volume and minimize its surface area?

15. Write the simplified formula for the surface area and volume of the optimal cylinder?

16. The volume of an optimal cylinder is 1125 cm³. Find its radius. Round your final answer to one decimal.

17. The minimum surface area of a cylinder is 1014π m². Determine the maximum volume of the cylinder
   a) In terms of pi
   b) Rounded to the nearest tenth.

18. The volume of a cylinder with optimal dimensions is 432π cm³. What is its height?

Solutions:

1. 144 m²
2. 32 m 3. P = 60 m; A = 225 m²
4a) 10 m by 10 m b) 40 m
5a) 130 m by 65 m b) P = 260 m
6a) 28 m b) 98 m²
c) 726 cm³
d) 486 cm²
7. 8 cm by 5 cm by 5 cm
8. 5 cm by 5 cm by 5 cm
9. 9261 cm³
10. 24 cm by 24 cm by 24 cm
11. 726 cm²
12a) 16 m, 16 m, 256 m² b) 125 km, 125 km, 15625 km²
c) 22 cm, 22 cm, 484 cm²
d) 2048 cm³, 512 cm³, 262144 cm³
e) 100 cm, 25 cm, 25 cm
f) 34 cm, 8.5 cm, 8.5 cm
13a) 32 m, 16 m, 512 m²
13b) 36 cm, 18 cm, 648 cm²
c) 50 km, 25 km, 1250 km²
d) 40 m, 20 m, 10 m
e) 60 m, 30 m, 15 m
f) h = d = 2r
15. SA = 6πr², V = 2πr³
16. 5.6 cm
17a) 4394π m³
17b) 13804.2 m³
18. 12 cm