1.6 Continuity

A Continuity

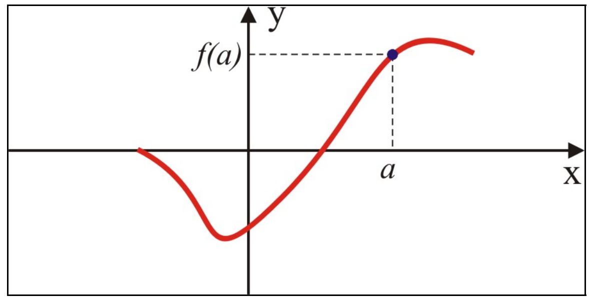
A function $y = f(x)$ is *continuous* at a number a if

$$\lim_{x \to a} f(x) = f(a)$$

Note: A function is continuous at a if the following three conditions are met:

1. $f(a)$ exists
2. $\lim_{x \to a} f(x)$ exists
3. $f(a)$ and $\lim_{x \to a} f(x)$ are equal.

Note: A function is continuous if the graph can be drawn *without lifting* the pen from paper.



B Discontinuity

If $y = f(x)$ is not continuous at a then we say:

- $y = f(x)$ is *discontinuous* at a or
- $y = f(x)$ has a discontinuity at a

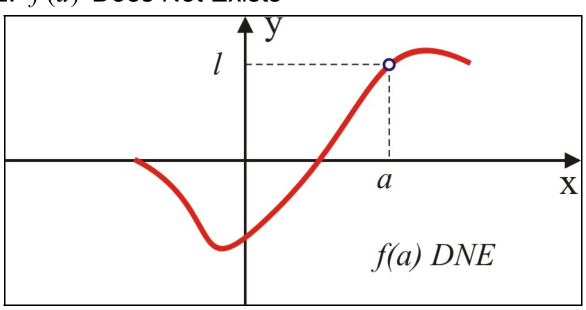
Note: There are three types of discontinuity:

a) *removable* or *point* discontinuity
b) *jump* discontinuity
c) *infinite* discontinuity (break)

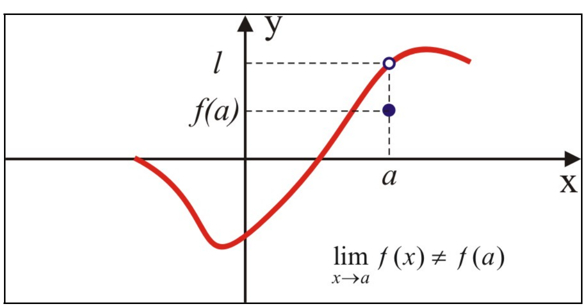
C Removable or Point Discontinuity

A function $y = f(x)$ has a *removable or point discontinuity* at a if:

1. $\lim_{x \to a} f(x)$ exists
2. $f(a)$ Does Not Exists



or $\lim_{x \to a} f(x) \neq f(a)$



Note: A removable or point discontinuity *can be removed* by redefining the function at a as

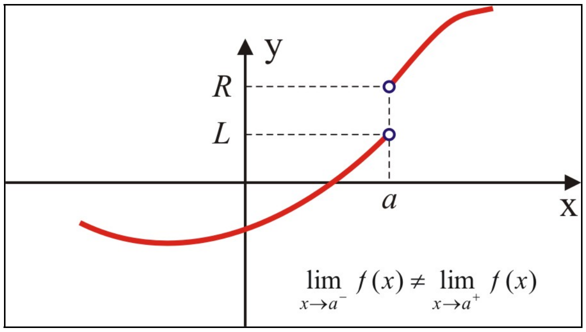
$$f(a) \overset{def}{=} \lim_{x \to a} f(x).$$

Ex 1. Redefine $y = f(x) = \dfrac{x^2 - 4}{x - 2}$ such that $y = f(x)$ is to be continuous everywhere (at any number). Graph the old and the new function.

D Jump Discontinuity

A function $y = f(x)$ has a *jump discontinuity* at a if the left-side and the right-side limits exist but they are not equal:

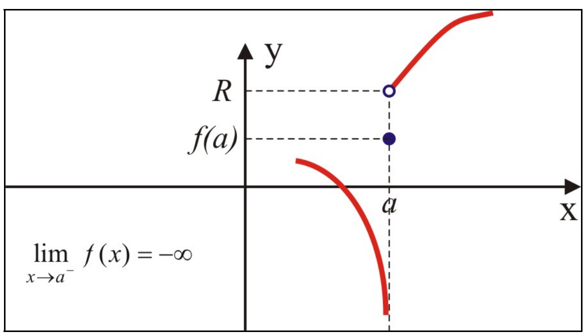
$$\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$$

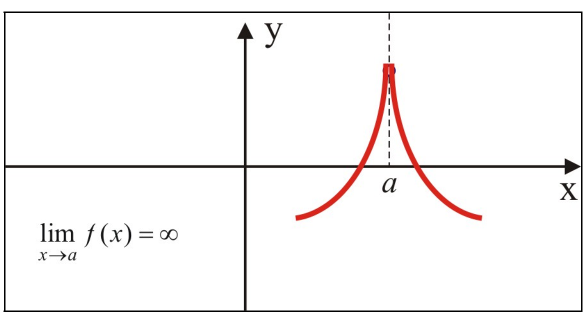


Ex 2. Analyze the continuity of the function $y = f(x) = \frac{|x-3|}{x-3}$ at $x = 3$. Graph the function to illustrate the situation.

E Infinite Discontinuity

A function $y = f(x)$ has an *infinite discontinuity* at a if at least one of the left-side or the right-side limits is *unbounded* (approaches to ∞ or $-\infty$).





To write $\lim_{x \to a} f(x) = \infty$ is better (more information is included) than to write $\lim_{x \to a} f(x)$ DNE.

Ex 3. Analyze the continuity of the function $f(x) = \frac{1}{x}$ at $x = 0$.

Ex 4. The function $y = f(x)$ is represented graphically in the figure on the right side.

Analyze the continuity of this function at:

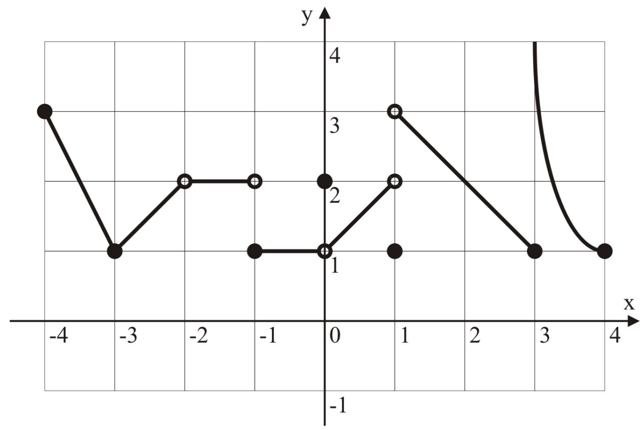
a) $x = -3$

b) $x = -2$

c) $x = -1$

d) $x = 0$

e) $x = 3$



F Elementary Functions
Elementary functions (polynomial, power, rational, trigonometric, exponential, and logarithmic) are *continuous* over their domain.

Ex 5. Analyze the continuity of the function:

$$f(x) = \begin{cases} x, & x < 0 \\ x^2, & 0 \le x \le 1 \\ x^3 + 1, & x > 1 \end{cases}$$

Ex 6. For what value of the constant c is the function

$$f(x) = \begin{cases} x + c, & x < 2 \\ cx^2 + 1, & x \ge 2 \end{cases}$$

continuous at any number (everywhere)?

Reading: Nelson Textbook, Pages 48-51
Homework: Nelson Textbook: Page 51 #4a, 5c, 7, 12, 15, 16, 17

