St. Paul's Sample Final Examination
MCV-4U
Time: 2.5 hours

Part A: Full solutions are not required, but part marks are earned for intermediate steps if the work is shown. Write your answer in the space provided.

1. Evaluate the following limits.

    a) $\lim\limits_{x \to -1} 2 - 3x^2$ _____

[2]

    b) $\lim\limits_{x \to -\infty} e^x$ _____

2. Determine the derivatives of the following functions. Do **Not** simplify your answers.

a) $y = \left(1 + \sqrt{x^3 + 3}\right)^5$ [3]

_____

b) $y = \dfrac{1 - \ln x}{\left(3x^2 - 5\right)^7}$ [3]

_____

c) $y = (\sin x)^6 \left(e^{2x}\right)$ [4]

_____

3. Are the following quantities Scalars (**S**), Vectors (**V**) or Meaningless (**M**)?

    a) 100 Nm of Torque is being applied down into the screw.   _____

    b) $\mathbf{a} \bullet (\mathbf{b} \times \mathbf{c})$ _____

    c) $\mathbf{b} - \mathbf{b}$ _____

4. Vectors **a** and **b**, with |**a**| = 13 and |**b**| = 7, **a** is horizontal and **b** is 60° *above* **a**.

    a) |**a** + **b**| _____

    b) The direction of |**a** + **b**| relative to **a**   _____

    c) **a**•**b**   _____

5. List Given vectors **a** = [1, -2, 3] and **b** $= 3\hat{i} - \hat{j} - 2\hat{k}$ determine the following.

   a) **a** • **b**   _____

   b) $\hat{b}$ _____

   c) the angle between **a** and **b**   _____

   d) a vector orthogonal to both **a** and **b**   _____

   e) Sketch vector **b**
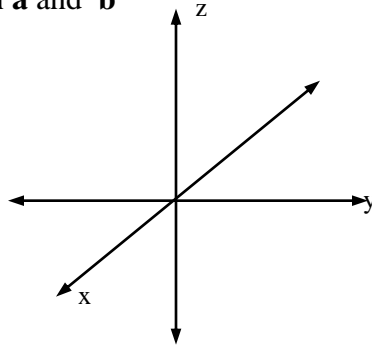
6. Given the graph of f', determine the following intervals:

Y=f′(x)
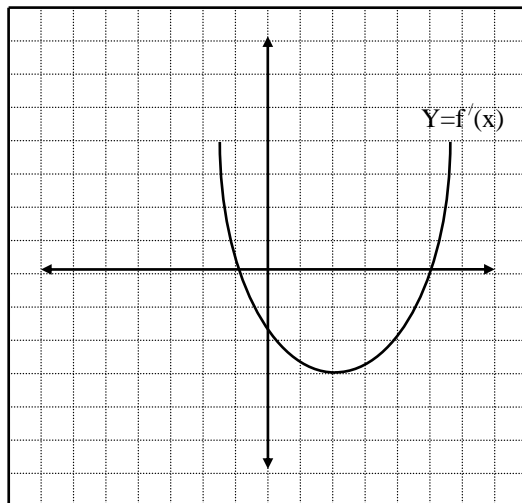
   a) the graph of f is decreasing   _____
[3]
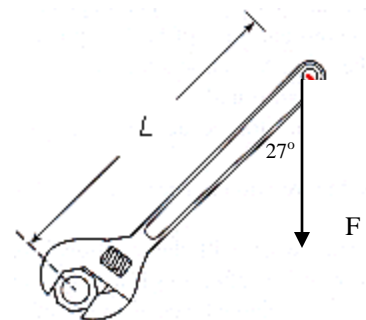
   b) the graph of f is concave down   _____

7. a) What is the magnitude of torque produced when a   _____
   300 N force is applied at an angle of 27° to a wrench
   that is 20 cm long?

   b) What is the direction of the torque?

   _____

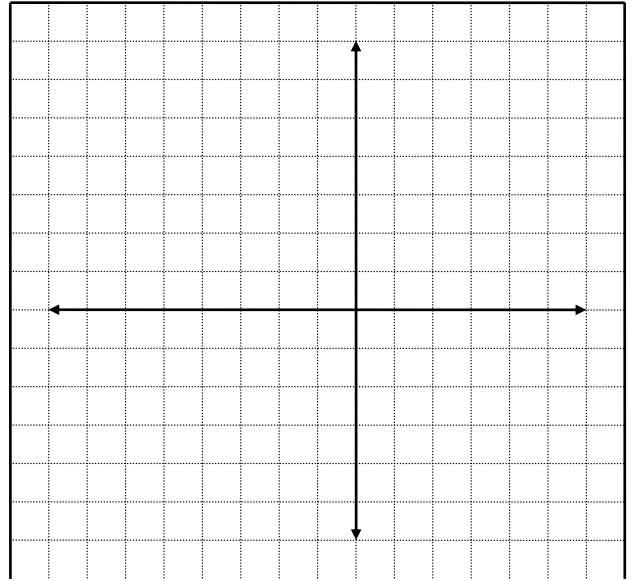8. A function, f, has all the properties listed below. Sketch the graph of $y = f(x)$.

a) $f(x)$ is an odd function, $f(x)$ has no y-intercept

b) $\lim_{x \to +\infty} f(x) = 2$,

[4]

c) $f(3) = 5,\ f'(3) = 0,\ f''(3) < 0$

d) $f''(x) < 0$ for $0 < x < 6$; and $f''(x) > 0$ for $x > 6$

9. A motorist driving on a straight and level road approaches an intersection. He sees a stop sign and applies his brakes. His car slows down in a way that his velocity, in meters per second, after **t** seconds is $v(t) = 80 - 10t^{\frac{3}{2}}$.

[1]   a) Determine the initial velocity of the car.

_____

[2]   b) How long does it take the car to stop?

_____

[1]   c) Determine the acceleration function of the car.

_____

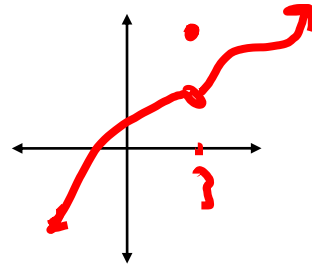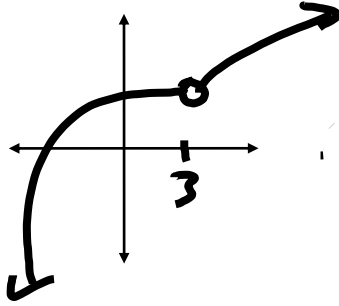[2]   e) Calculate the average acceleration of the car while braking.

_____

10. Provide a sketch of a function $f(x)$ that is discontinuous at x = 3 because…

    a) $f(3)$ does not exist

                                b) $\lim\limits_{x\to 3} f(x) \neq f(3)$

[2]

11. Determine the following limits **exactly** if they exist, and state if they do not.

    a) $\lim\limits_{x\to 2} \dfrac{x^3 - 8}{2x^2 + x - 10}$ _____
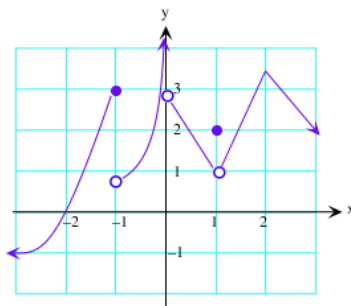
    b) $\lim\limits_{x\to\infty} \dfrac{9x^4 - 3x^7 + 6}{2x - x^4 + 6x^7}$ _____

    c) $\lim\limits_{x\to 5^-} \dfrac{5x - x^2}{|x - 5|}$ _____

    d) $\lim\limits_{x\to 0}(1 + x)^{\frac{2}{x}}$ _____

    e) $\lim\limits_{h\to 0} \dfrac{(x + h)^{12} - x^{12}}{h}$ _____

    f) Given the graph of $f(x)$, evaluate $\lim\limits_{x\to 2} f(x)$ _____

    g) Given $f(x) = \begin{cases} 2 - x & x > 0 \\ \sqrt{x} + 4 & x = 0 \\ x^2 + 2 & x < 0 \end{cases}$, evaluate $\lim\limits_{x\to 0} f(x)$ _____

PART B: Full solutions are required. Answer in the spaces provided.

1.   Determine the following limit exactly. Show all work.   [3]

$$\lim_{x \to 4} \frac{\dfrac{1}{x} - \dfrac{1}{2\sqrt{x}}}{x - 4}$$

2. The value of a $1000 investment earning 6% compounded annually is given by

   $V = 1000(1.06)^n$, where n is the number of years the money remains invested.

a)   What is the average rate of change of value for the first 3 years of the investment?

b)   How fast the investment is growing at ten years.

[4]

3. Match each given function to its derivative below. Give one reason for each choice.
    Sketch the unmatched derivative.

i)   ii)   iii)

A)   B)   C)
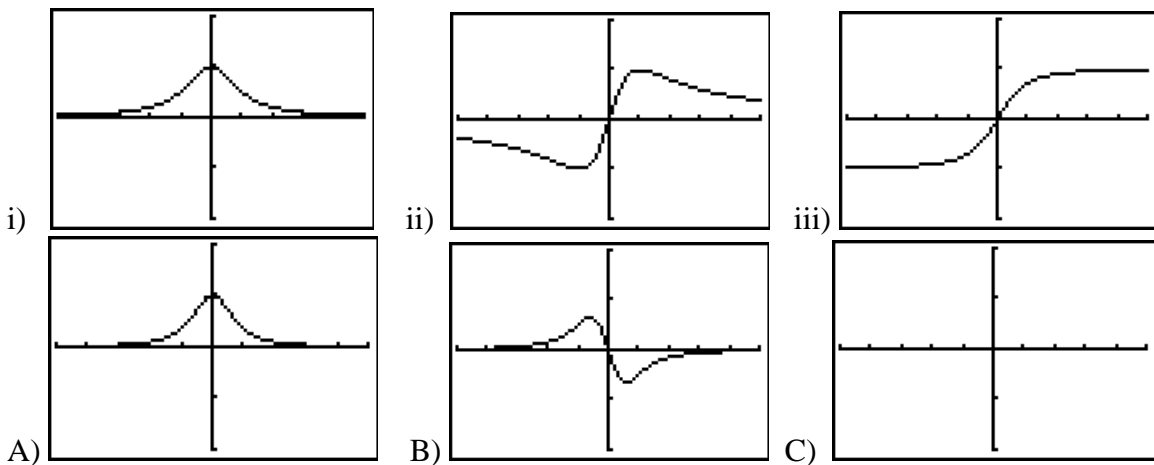
[5]

4. Determine and simplify the derivative of $f(x) = \dfrac{3}{1-2x}$ from first principles.

[5]

5. A piece of string 100cm long is to be cut into two pieces. One piece will be bent into a circle and the other will be bent into a square. Where should the string be cut in order to minimize the total area of the two figures. Verify that your answer is indeed a minimum. (Answer to two decimal places)

[6]

6. Examine the graph of the function $y = A^x$ and
   its derivative. Assume each y-intercept
   crosses at the line on the graph paper.   [5]
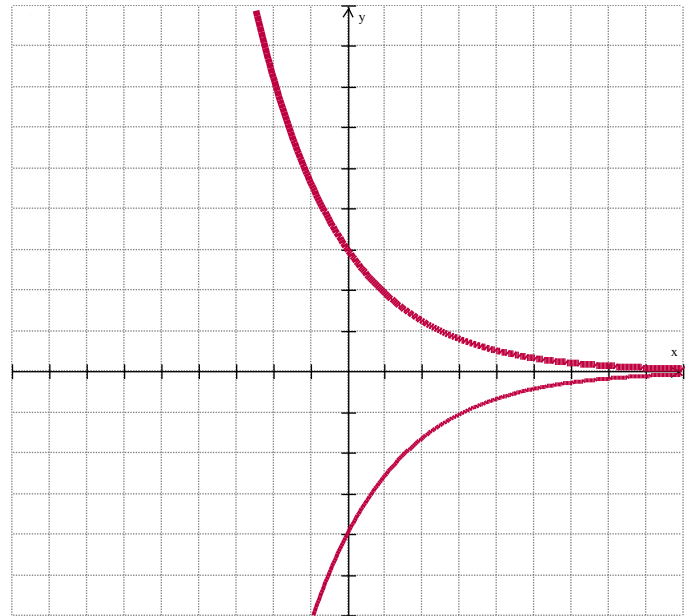
a) Which is the function? How do you know?

b) What is the scale on the y-axis?
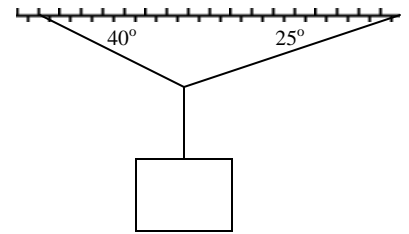
c) What is the equation of the tangent at x = 0?

d) What is the value of A?   Explain.

7. A plane travels 300 km at (a bearing of) 120° then travels 250 km at 200°. Give a
    single vector for the direct route flight.

[4]

8. A 100 kg weight is held from the ceiling by two cables, as shown. If the system is perfectly balanced, determine the tension (One decimal place) in each cable.

[5]

9. Find the scalar equation of the plane that contains the point (2, -1, 5) as well as the line $<x, y, z> = <1, 2, 3> + t<3, 4, -3>$.

[5]

10. Solve and classify each intersection.   [10]

a) $[x,y,z] = [0,-8,4] + t[3,1,-1]$ and $\dfrac{x-3}{1} = \dfrac{y+7}{-2} = \dfrac{z-5}{4}$

b) $x + 2y + 3z + 4 = 0$
    $x - y - 3z - 8 = 0$
    $x + 5y + 9z + 16 = 0$

11. Analyze the function $y = \dfrac{x^2 + 1}{(x-1)^2}$, under the headings: Domain, Intercepts,

Asymptotes, Increasing/Decreasing, Maximum/Minimum, Concavity and Points of Inflection.   Then sketch the curve.

[13]