Unit 6: Practice Test

① a) Scalar (no direction) b) Vector

② a) $\vec{WU}$ b) $\vec{PQ}+\vec{TU}$ $=\vec{TS}+\vec{SW}$ $=\vec{TW}$ c) $\vec{UR}$ d) $\vec{QW}+\vec{US}+\vec{TV}$ $=\vec{PU}+\vec{US}+\vec{TV}$ $=\vec{PS}+\vec{TV}$ $=\vec{PS}+\vec{SR}$ $=\vec{PR}$

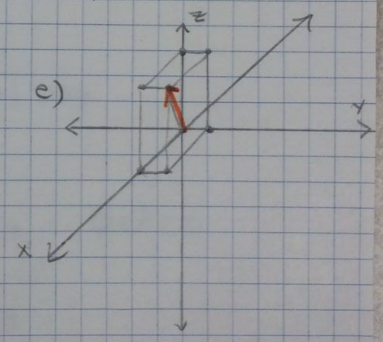
③ $\vec{a}=(2,-1,3)$ $\vec{b}=3\hat{i}-\hat{j}+\hat{k}=(3,-1,1)$

a) $3\vec{a}-\vec{b}$
$=3(2,-1,3)-(3,-1,1)$
$=(6,-3,9)-(3,-1,1)$
$=(3,-2,8)$

b) $|\vec{b}|=\sqrt{(3)^2+(-1)^2+(1)^2}$ $=\sqrt{11}$

c) $\hat{a}=\frac{1}{|\vec{a}|}\vec{a}$ $|\vec{a}|=\sqrt{14}$
$=\frac{1}{\sqrt{14}}(2,-1,3)$
$=\left(\frac{2}{\sqrt{14}},\frac{-1}{\sqrt{14}},\frac{3}{\sqrt{14}}\right)$

e)



d) $\vec{a}\cdot\vec{b}=(2,-1,3)\cdot(3,-1,1)$
$=6+1+3$
$=10$

f) $\cos\beta=\frac{b}{\sqrt{a^2+b^2+c^2}}$

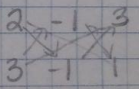
$\cos\beta=\frac{-1}{\sqrt{11}}$ ← y-coord of $\vec{b}$

↑ w/ y-axis

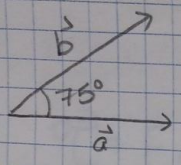
$\beta\doteq 108^\circ$

g) Orthogonal is Perpendicular

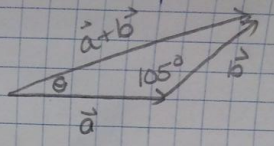
$\vec{a}\times\vec{b}$
$=(2,7,1)$



h) $(\vec{a}\times\vec{b})\cdot\vec{b}$
$=(2,7,1)\cdot(3,-1,1)$
$=(6,-7,1)$

4) 

a) $|\vec{a}+\vec{b}|$



$|\vec{a}+\vec{b}|^2 = 10^2+25^2-2(10)(25)\cos 105°$

$|\vec{a}+\vec{b}| \doteq 29$

b) $\frac{\sin\theta}{25} = \frac{\sin 105°}{29}$

$\theta \doteq 56°$

c) $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$= (10)(25)\cos 75°$

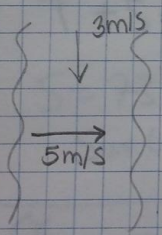
$\doteq 65$

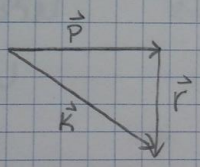
d) $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

$= (10)(25)\sin 75°$

$\doteq 241.5$

e) use right-hand rule: OUT of page

5) 



$\vec{p}+\vec{r}=\vec{k}$

$|k| = \sqrt{|\vec{p}|^2+|\vec{r}|^2}$

$= \sqrt{3^2+5^2}$

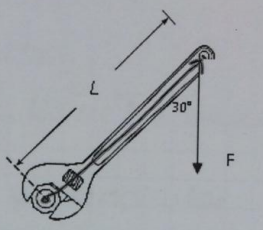
$\doteq 5.8$ m/s

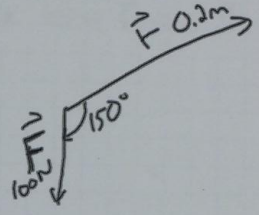
1. How much torque exists when a 100 N force is applied at and angle of 30° to a wrench that is 20 cm long? [3]

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau} = |0.2||100| \sin 150° \hat{n}$

$= 10 N \cdot m \ (\hat{n})$





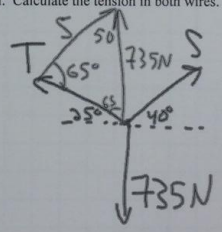
May 16-9:12 AM

A weight of 75 kg is suspended from two wires. One wire makes an angle of 25° with the horizont The other wire makes an angle of 40° with the horizontal. Calculate the tension in both wires.

$\vec{T} + \vec{S} = 735N$

$S = 735N$

SINE LAW



OR

$T_x = S_x$ ①

$T \cos 25 = S \cos 40$

$T = S\left(\frac{\cos 40}{\cos 25}\right)$ ... ②

$T_y + S_y = 735$

$T \sin 25 + S \sin 40 = 735$

May 16-9:17 AM

COMM

a) $\vec{u} = k\vec{v}$ $\vec{u}, \vec{v}$ are collinear

$\vec{u}, \vec{v}$ are Parallel

$\vec{u}, \vec{v}$ may be 2D or 3D vectors

b) $\vec{u} \cdot \vec{v} = 0$ $\vec{u}, \vec{v}$ are perpendicular

the angle b/w $\vec{u}$ and $\vec{v}$ is 90°

c) $\vec{u} = (x, 0, z)$

↑ always 0

1. Any vector in the x-y plane can be written as a combination of the component vectors **i** and **j**. For example **a** = (2, -4) would be **a** = 2**i** - 4**j**. Every vector in the x-y plane can also be written as a combination of **v** = (1, 1) and **w** = (0, 1)

a) Find scalars k and m, such that **a** = k**v** + m**w**.

b) Show that **any vector** can be written as a combination of **v** = (1, 1) and **w** = (0, 1).

c) This is not true for any two vectors. Can every vector be written as a combination of **s** = (0.5, -2) and **t** = (-2, 8)? Explain.

a) $\langle 2,-4\rangle = k\langle 1,1\rangle + m\langle 0,1\rangle$

$2 = k+0 \rightsquigarrow k=2$

$-4 = k+m \quad m=-6$

b) $\vec{v}=\langle x,y\rangle$

$\langle x,y\rangle = k\langle 1,1\rangle + m\langle 0,1\rangle$

$x = k+0$ $\quad k = x$

$y = k+m \quad m = y-x$

May 16-8:57 AM

2. Prove Lagrange's Identity $|\vec{u}\times\vec{v}|^2 = |\vec{u}|^2|\vec{v}|^2 - (\vec{u}\bullet\vec{v})^2$

$|\vec{u}\times\vec{v}| = |\vec{u}||\vec{v}|\sin\theta$

$|\vec{u}\times\vec{v}|^2 = |\vec{u}|^2|\vec{v}|^2\sin^2\theta$

$L.S. = |\vec{u}|^2|\vec{v}|^2\sin^2\theta$

$(\vec{u}\cdot\vec{v}) = |\vec{u}||\vec{v}|\cos\theta$

$(\vec{u}\cdot\vec{v})^2 = |\vec{u}|^2|\vec{v}|^2\cos^2\theta$

$R.S. = |\vec{u}|^2|\vec{v}|^2 - |\vec{u}|^2|\vec{v}|^2\cos^2\theta$

$= |\vec{u}|^2|\vec{v}|^2(1-\cos^2\theta)$

$= |\vec{u}||\vec{v}|\sin^2\theta$

$= L.S.$

May 16-9:05 AM