Lesson 4: Vectors in 3-Space

PART A: Forms of Vectors

1. Geometric: As a directed line segment, *(length corresponds to magnitude, angle corresponds to direction)*

Magnitude and Direction: Size at a given direction,

Ex: 100 km at N38°W, or 12 N at 5° from the horizontal,

10 m/s at 16° (*North is 0°*) (5, 15°) *polar coordinates* (r, Θ)

2. Algebraic:

a) Ordered Pair Notation

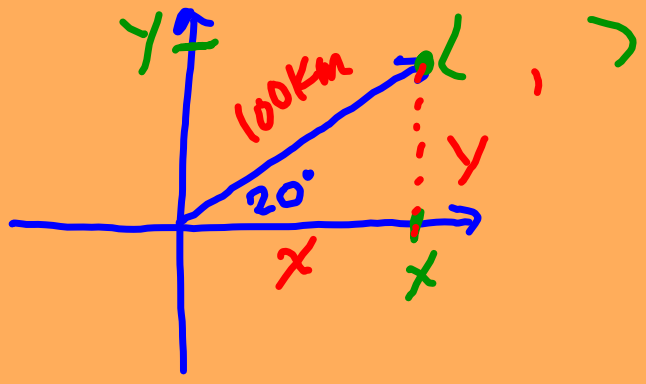
Written as an ordered pair (or triplet) of coordinates for the tip of the vector when the tail lies at the origin.

i.e. if **u** = <a, b> then **u**= $\vec{OP}$, where O is the origin and P is (a, b) or (a, b, c) for 3-D

b) Unit Vector Notation

Where **u**= $\vec{OP}$, and **u** = $a\hat{i} + b\hat{j}$ or **u** = $a\hat{i} + b\hat{j} + c\hat{k}$ for 3-D vectors where $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors, along the x, y and z axis respectively.

Ex 1: Express 100 km at 20° from the horizontal in algebraic form



$\langle \quad , \quad \rangle$

$\cos 20 = \frac{x}{100}$

$x = 100\cos 20°$

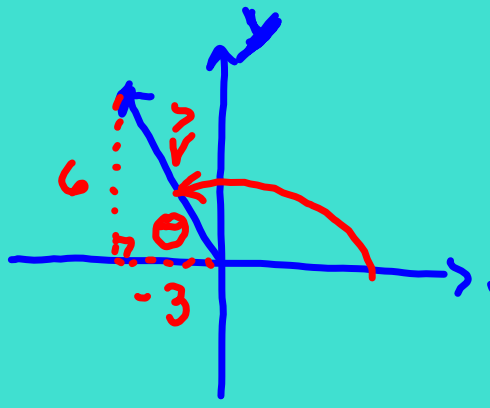
$\doteq 94.0\text{km}$

$\sin 20 = \frac{y}{100}$

$y \doteq 34\text{km}$

$\langle 94, 34 \rangle$

Ex 2: a) give the magnitude and direction of vector <-3, 6>
b) Generalize for <a, b>



$|\vec{v}| = 6.7$ units

$\theta = 63°$

$\therefore \vec{v}$ is 6.7 units @ 117° from the horizontal

OR 27° W of N

b) $|\langle a, b\rangle| = \sqrt{a^2 + b^2}$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

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Ex 3: Give the algebraic vectors for $\hat{i}, \hat{j}$ and $\hat{k}$.

$\hat{i} = \langle 1, 0, 0 \rangle$

$\hat{j} = \langle 0, 1, 0 \rangle$

$\hat{k} = \langle 0, 0, 1 \rangle$

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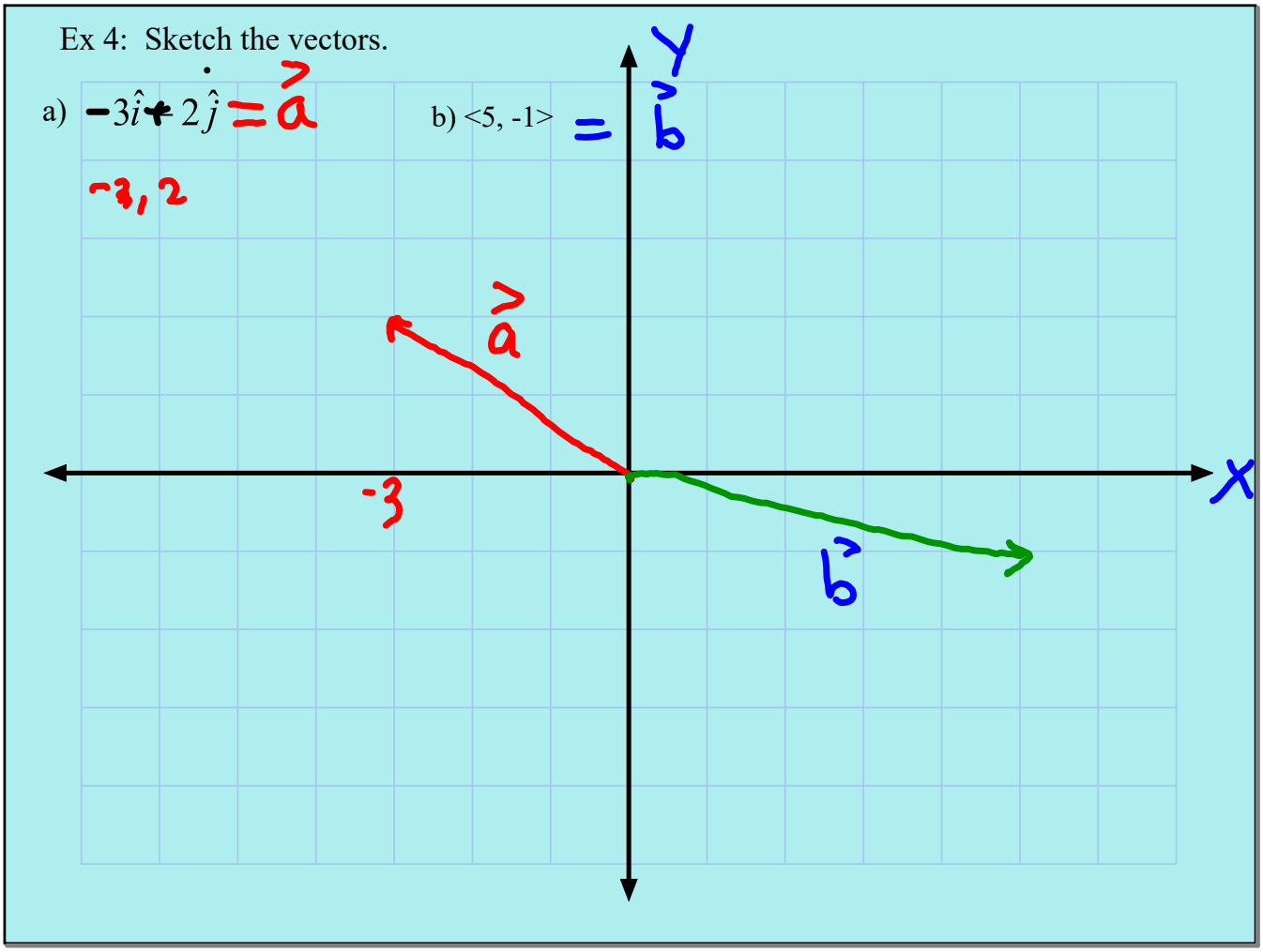
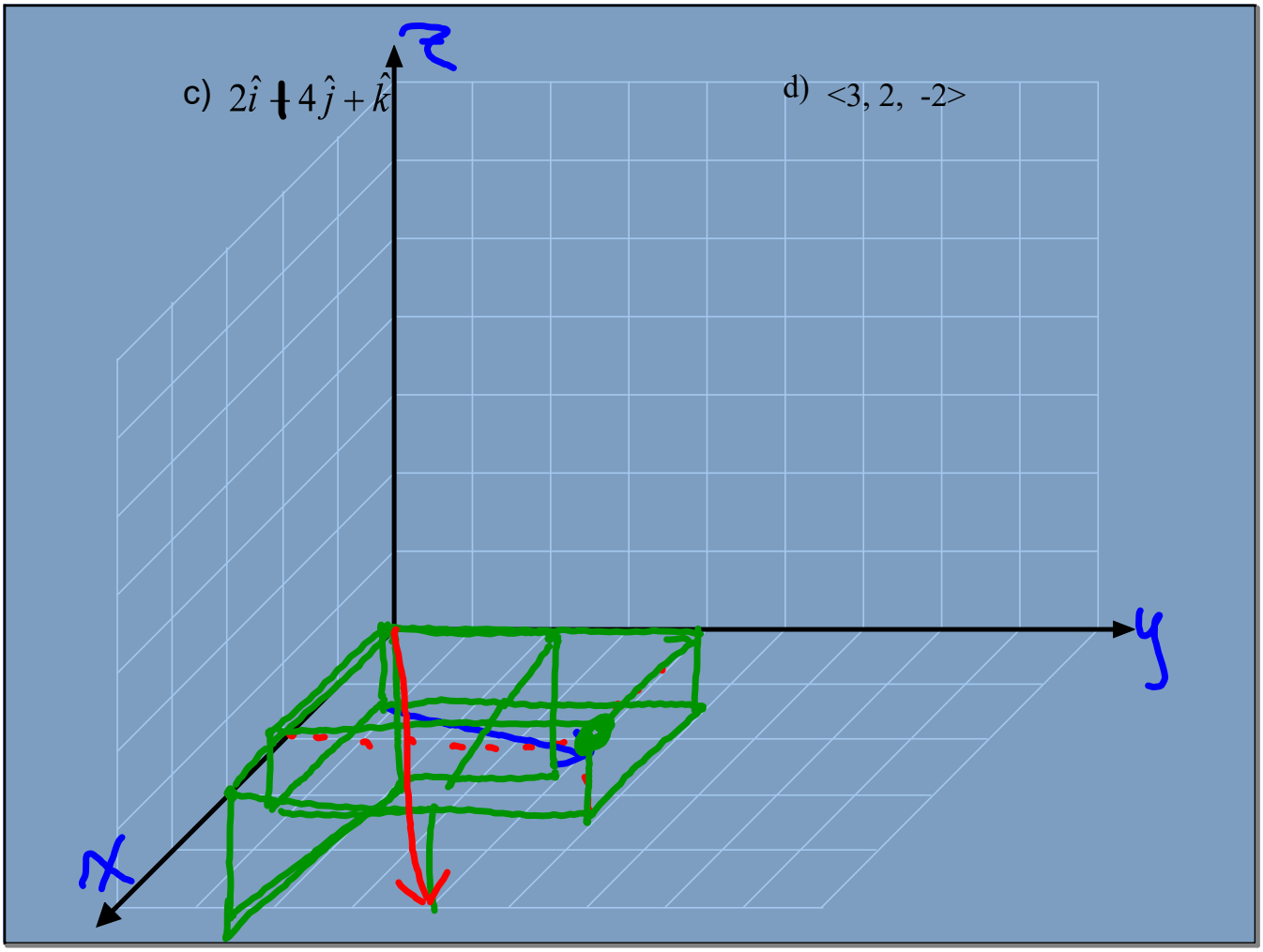
Ex 4: Sketch the vectors.

a) $-3\hat{i} + 2\hat{j} = \vec{a}$

b) <5, -1> = $\vec{b}$





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Ex 5: Determine the magnitude of the vector <3, -4, 12>.

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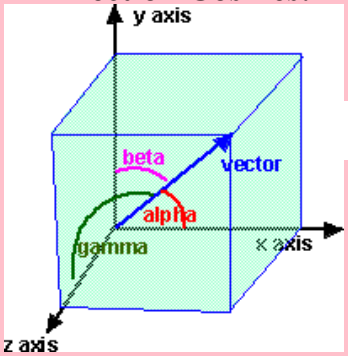
PART B: 3-Space Rules

3-D Pythagorean Theorem $|u| = \sqrt{u_x^2 + u_y^2 + u_z^2}$

Direction Angles: The set of angles α, β, and γ, a vector<a, b, c> makes with the positive coordinate axes. (x, y and z, respectively) α, β, and γ all lie between zero and 180°

Direction Cosines: The cosines of the direction angles



$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$$

$$\cos\alpha = \frac{a}{|\vec{u}|}$$

$$\cos\beta = \frac{b}{|\vec{u}|}$$

$$\cos\gamma = \frac{c}{|\vec{u}|}$$

and

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Ex 6: Determine the direction angles and cosines of <3, -2, 7>.

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Ex 7: A vector makes an angle of 30^{o} with the x-axis and 120^{o} with the y-axis, what angle does it make with the z-axis?

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PART C: Problems.

Example 1: Given $\mathbf{u} = \langle 2, -3, 5 \rangle$ and $\mathbf{v} = \langle 4, -1, -3 \rangle$ determine the following vectors.

a) $3\mathbf{u}$ b) $2\mathbf{u} - 3\mathbf{v}$ c) $\mathbf{v} + \hat{i}$

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Example 2: Prove that points A(-2, 5), B(4, 3) and C(-5, 6) are collinear.

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Example 3: Prove that $|k\vec{v}| = |k| \times |\vec{v}|$

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