

①

## Lesson 4C Examples:

$$1) f(x) = \begin{cases} 3-x, & x < 2 \\ x^2-3, & x > 2 \end{cases} \quad \lim_{x \rightarrow 2} f(x) = 1$$

$$\begin{aligned} \lim_{x \rightarrow 2^-} 3-x & \quad \lim_{x \rightarrow 2^+} x^2-3 \\ = 1 & \quad = 4-3 = 1 \end{aligned}$$

$$2) a) \lim_{x \rightarrow 9} \frac{\frac{1}{\sqrt{x}} - \frac{1}{3}}{x-9} \quad \begin{aligned} \text{let } u &= \sqrt{x} \\ u &= x^{1/2} \\ u^2 &= x \end{aligned}$$

$$= \lim_{x \rightarrow 9} \frac{3-\sqrt{x}}{3\sqrt{x}} \cdot \frac{1}{x-9}$$

$$= \lim_{x \rightarrow 9} \frac{3-u}{3u} \cdot \frac{1}{u^2-9}$$

$$= \lim_{x \rightarrow 9} \frac{3-u}{3u} \cdot \frac{1}{(u-3)(u+3)}$$

$$= \lim_{x \rightarrow 9} \frac{-1}{3u(u+3)}$$

$$= \lim_{x \rightarrow 9} \frac{-1}{3\sqrt{x}(\sqrt{x}+3)}$$

$$= \frac{-1}{9(6)3+9}$$

$$= \frac{-1}{54}$$

$$2b) \lim_{x \rightarrow 3} \frac{\sqrt{2x+1} - \sqrt{7}}{6x - 2x^2}$$

$$= \lim_{x \rightarrow 3} \frac{2x+1-7}{2x(3-x)(\sqrt{2x+1} + \sqrt{7})}$$

$$= \lim_{x \rightarrow 3} \frac{2x-6}{2x(3-x)(\sqrt{2x+1} + \sqrt{7})}$$

$$= \lim_{x \rightarrow 3} \frac{-2(-x+3)}{2x(3-x)(\sqrt{2x+1} + \sqrt{7})}$$

$$= \lim_{x \rightarrow 3} \frac{-1}{x(\sqrt{2x+1} + \sqrt{7})}$$

$$= \frac{-1}{3(\sqrt{7} + \sqrt{7})}$$

$$= \frac{-1}{6\sqrt{7}}$$

$$c) \lim_{x \rightarrow 1^+} \frac{x+1}{(1-x)(1+x)}$$

$$= \lim_{x \rightarrow 1^+} \frac{1}{1-x} \quad x=1.1 \quad x=1.01$$

$f(x) = -10, \quad f(x) = -100$

$$= -\infty$$

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$$d) \lim_{x \rightarrow 1^+} \frac{x-1}{(1-x)(1+x)}$$

$$= \lim_{x \rightarrow 1^+} \frac{-(1-x)}{(1-x)(1+x)}$$

$$= \lim_{x \rightarrow 1^+} \frac{-1}{1+x} \quad \begin{array}{ccc} x=1.1 & x=1.01 & 1.00001 \\ f(x)=-2.1 & -0.5 & -0.5 \end{array}$$

$$= -\frac{1}{2}$$

$$e) \lim_{x \rightarrow 1^-} \frac{x+1}{(1-x)(1+x)}$$

$$= \lim_{x \rightarrow 1^-} \frac{1}{1-x} \quad \begin{array}{ccc} x=0.9 & x=0.99 \\ f(x)=10 & =100 \end{array}$$

$$= \infty$$

Ex 4.5 (3x+2)

$$g) \lim_{x \rightarrow 3^+} \frac{x|x-3|}{x-3}$$

$$= \lim_{x \rightarrow 3} \frac{x(x-3)}{x-3}$$

$$= 3$$

$$h) \lim_{x \rightarrow 3} \frac{x|x-3|}{x-3}$$

$$= \text{DNE}$$

$$f(x) = \begin{cases} \frac{x(x-3)}{x-3}, & x \geq 3 \\ \frac{-x(x-3)}{x-3}, & x < 3 \end{cases}$$

$$\lim_{x \rightarrow 3^-} \frac{x|x-3|}{x-3}$$

$$= \lim_{x \rightarrow 3} \frac{-x(x-3)}{x-3}$$

$$= -3$$

x	f(x)
3.1	

$$\begin{aligned} 3) \lim_{x \rightarrow 8^-} \sqrt{x+8} \\ = 4 \end{aligned}$$

x	7.9	7.99
f(x)	3.98	3.99

$$\begin{aligned} 4) \lim_{x \rightarrow 0} 2\sqrt{x} \\ = 2\sqrt{0} \\ = 0 \end{aligned}$$

\*But, x is an endpoint at 0, so  $\lim_{x \rightarrow 0} = \text{DNE}$

$$5) a) x = -1, 0,$$

$$b) x = -1, 0, 1$$

c)  $|x| = \pm 1$  : jump point  
 $x = 0$  : VA left  
Hole right