

Lesson 6: Review & APPS

Ex 1: The cost function for producing x items is given by
 $C(x) = 10\,000 + 5x + 0.01x^2$.

- a) Find the cost of producing 250 items.
 b) Find the marginal cost of producing 200 items. (derivative)
 c) What is the actual cost of producing the 201st item?
 d) What is the **average cost** for producing 200 items?

a) $C(250) = 10000 + 5(250) + 0.01(250)^2$
 $C(250) = 18075$

b) $C'(x) = 5 + 0.02x$
 $C'(200) = 9$ / item

c) $C(201) - C(200)$
 $= 11409.01 - 11400$
 $= 9.01$

d) $C_{av}(200) = \frac{C(200)}{200}$
 $= \frac{11400}{200}$
 $C_{av}(200) = 57$ / item

$C(1) = 110$
 $100 + 10$
 $C(2) = 120$
 $60 / \text{item}$
 $\frac{120}{2}$

Feb 22-9:29 AM

Ex 2: If a tank holds 60L of water and takes 60 minutes to drain from the bottom, then Torricelli's Law gives the volume, V of water remaining in the tank after t minutes.

$$V = 60 \left(1 - \frac{t}{60} \right)^2 \quad 0 < t < 60$$

- a) How much water is left after 30 minutes?

$$V(30) = 15L$$

- b) What is the average rate of flow for the first 30 minutes?

$$\frac{V(30) - V(0)}{30 - 0} = \frac{15 - 60}{30} = -1.5 \text{ L/min}$$

- c) How fast is it water flowing out after 30 minutes?

$$V'(t) = 120 \left(1 - \frac{t}{60} \right) \left(-\frac{1}{60} \right)$$

$$V'(30) = -1 \text{ L/min}$$

Feb 22-9:31 AM

Ex 3: A particle slides along the x-axis with its position (s in m) given as a function of time (t in s) where $s(t) = t^2 - 8t + 7$.

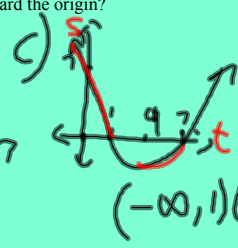
a) When is the particle at the origin? $s(t) = 0$

b) When is the particle at rest? $v = 0$

c) When is the particle moving toward the origin?

a) $t^2 - 8t + 7 = 0$
 $0 = (t-1)(t-7)$
 $t = 1, 7$

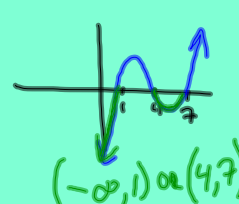
b) $s(t) = t^2 - 8t + 7$
 $v(t) = 2t - 8$
 $t = 4 \text{ sec}$

c) 

OR MOVING TOWARD THE ORIGIN
 $s(t) > 0 \wedge v(t) < 0$
 $s(t) < 0 \wedge v(t) > 0$

MOVING TOWARD ORIGIN
 $s(t) \cdot v(t) < 0$

$s(t) \cdot v(t) < 0$
 $(t^2 - 8t + 7)(2t - 8) < 0$
 $(t-1)(t-7)(t-4) < 0$
 $t = 1, 4, 7$



Feb 22-9:34 AM

12 $f(x) = ax^2 + bx + c$ $f'(x) = 2ax + b$

$f(2) = 19 \rightarrow 4a + 2b + c = 19$ ①

$f(-1) = -8 \rightarrow a - b + c = -8$ ②

$f'(-1) = 0 \rightarrow -2a + b = 0$ ③

$b = 2a$ ① + ③

$-a + c = -8$

$7a + c = 19$

$8a = 27$

$a = 3$ $c = -5$ $b = 6$

OR

$f(x) = a(x-h)^2 + k$

$f(x) = a(x+1)^2 + 8$

Jul 11-9:36 AM

83#9c) $y = \sqrt{3x^3}$ $P(3, 9)$

$(3^{\frac{1}{2}})$
 $(\sqrt{3})$

$$y = \sqrt{3} X^{\frac{3}{2}}$$

$$y' = \frac{3\sqrt{3}}{2} X^{\frac{1}{2}}$$

$$y'(3) = \frac{3\sqrt{3}}{2} (\sqrt{3})$$

$$= 4.5$$

$$y = 4.5x + b$$

$$9 = 4.5(3) + b$$

$$b = -4.5$$

$$y = 4.5x - 4.5$$

$$y = (3x^3)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}(3x^3)^{-\frac{1}{2}} \cdot 9x^2$$

$$y'(3) = \frac{1}{2}(81)^{-\frac{1}{2}} \cdot 9(9)$$

$$= \frac{1}{2}(\frac{1}{9}) \cdot 81$$

$$= 4.5$$

$$2y = 9x - 9$$

$$0 = 9x - 2y - 9$$

Feb 29-8:45 AM

11. TANGENT TO $y = 3x^{-\frac{1}{3}}$ // to $x + 16y + 3 = 0$

$$y' = -x^{-\frac{4}{3}}$$

SAME SLOPE

$$16y = -x - 3$$

$$y = -\frac{1}{16}x - \frac{3}{16}$$

$$m = -\frac{1}{16}$$

$$\therefore -\frac{1}{16} = -\frac{1}{x^{\frac{4}{3}}}$$

$$(16)^{\frac{3}{4}} = (x^{\frac{4}{3}})^{\frac{3}{4}}$$

$$8 = x$$

Feb 29-8:52 AM

$$\underline{\underline{26}} \text{ P89 } \sqrt{x} + \sqrt{y} = 1 \quad (a, b) \quad m_p = -\sqrt{\frac{b}{a}}$$

$$\sqrt{y} = 1 - \sqrt{x} \quad \rightarrow \quad \sqrt{b} = 1 - \sqrt{a}$$

$$y = (1 - \sqrt{x})^2$$

$$y = 1 - 2\sqrt{x} + x$$

$$y' = -x^{-\frac{1}{2}} + 1$$

$$y'(a) = -\frac{1}{\sqrt{a}} + 1$$

$$= 1 - \frac{1}{\sqrt{a}}$$

$$= \frac{\sqrt{a} - 1}{\sqrt{a}}$$

$$= -\frac{\sqrt{b}}{\sqrt{a}}$$

$$= -\sqrt{\frac{b}{a}}$$

Feb 29-8:56 AM

$$f(x) = 5\sqrt{x} - \frac{7}{x^3} + 8\sqrt[4]{x^5}$$

$$f(x) = 5x^{\frac{1}{2}} - 7x^{-3} + 8x^{\frac{5}{4}}$$

$$f'(x) = 2.5x^{-\frac{1}{2}} + 21x^{-4} + 10x^{\frac{1}{4}}$$

Feb 29-10:00 AM

$$92\#5$$

$$f(x) = 2x^4$$

$$f'(x) = 8x^3$$

$$\text{Set } f'(x) = 1$$

$$1 = 8x^3$$

$$\frac{1}{8} = x^3$$

$$x = \frac{1}{2}$$

$$\left(\frac{1}{2}, \frac{1}{8}\right)$$

$$m = 1$$

$$y = x + b$$

$$\frac{1}{8} = \frac{1}{2} + b$$

$$b = -\frac{3}{8}$$

$$y = x - \frac{3}{8}$$

Feb 29-10:04 AM

$$1c) \quad \underline{\underline{\neq 50\%}}$$

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← More ←

Feb 29-10:40 AM