Chapter 2 Permutations

Chapter 2 Prerequisite Skills

Chapter 2 Prerequisite Skills Question 1 Page 62

a) The list 0.5, 0.24, 0.718, 0.039 in order from least to greatest is 0.039, 0.24, 0.5, 0.718.

b) The list 3.78, 3.078, 3.0078 in order from least to greatest is 3.0078, 3.078, 3.78.

c) The list $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}$ in order from least to greatest is $\frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$.

d) First, rewrite the list $\frac{5}{8}, \frac{3}{4}, \frac{5}{6}, \frac{7}{12}$ using a common denominator of 24.

$\frac{5}{8} = \frac{15}{24}$ $\frac{3}{4} = \frac{18}{24}$ $\frac{5}{6} = \frac{20}{24}$ $\frac{7}{12} = \frac{14}{24}$

Then, the list in order from least to greatest is $\frac{7}{12}, \frac{5}{8}, \frac{3}{4}, \frac{5}{6}$.

Chapter 2 Prerequisite Skills Question 2 Page 62

a) $0.275 \times 100 = 27.5\%$

b) $4.9 \times 100 = 490\%$

c) $125.62 \times 100 = 12\ 562\%$

d) $2 \div 5 = 0.4$
$0.4 \times 100 = 40\%$

e) $57 \div 12 = 4.75$
$4.75 \times 100 = 475\%$

Chapter 2 Prerequisite Skills Question 3 Page 62

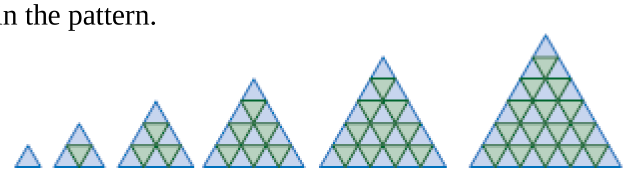
First, rewrite the sequence $\frac{2}{8}, \frac{7}{12}, \frac{33}{36}, \frac{20}{16}$ using a common denominator of 72.

$\frac{18}{72}, \frac{42}{72}, \frac{66}{72}, \frac{90}{72}$

Since the numerators increase by 24, the next number in the sequence is $\frac{90+24}{72} = \frac{114}{72}$, or $\frac{57}{36}$.

Chapter 2 Prerequisite Skills Question 4 Page 62

a) Starting with one triangle, add an increasing number of odd triangles to form a larger triangle in the pattern.



b) Starting with 12, subtract 3 continuously to get new terms in the pattern.
12, 9, 6, 3, 3 – 3 = 0, 0 – 3 = –3, –3 – 3 = –6, …

c) Starting with the expression $n - 2$, subtract 1 continuously to get new terms in the pattern.
$n - 2, n - 3, n - 4, n - 4 - 1 = n - 5, n - 5 - 1 = n - 6, n - 6 - 1 = n - 7, \ldots$

d) Starting with $\frac{1}{2}$, multiply the denominator by 2 continuously to get new terms in the pattern.

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8\times 2} = \frac{1}{16}, \frac{1}{16\times 2} = \frac{1}{32}, \frac{1}{32\times 2} = \frac{1}{64}, \ldots$$

Chapter 2 Prerequisite Skills Question 5 Page 62

Answers may vary.

a) If you view the diagrams as stairs, start with 2 steps then add 1 step continuously to get the next diagram in the pattern. 2, 3, 4, …

Starting with a perimeter comprised of 6 line segments, add 2 line segments continuously to get the next diagram in the pattern. 6, 8, 10, …

b) The first sequence extended is 2, 3, 4, 5, 6, …. The second sequence extended is 6, 8, 10, 12, 14, ….

Chapter 2 Prerequisite Skills Question 6 Page 62

a)
$$\begin{aligned}(12)(11)(10) - (9)(8)(7) &= 1320 - 504 \\ &= 816\end{aligned}$$

b)
$$\begin{aligned}\frac{5\times 4\times 3\times 2\times 1}{3\times 2\times 1} + \frac{6\times 5\times 4}{4\times 3} &= \frac{120}{6} + \frac{120}{12} \\ &= \frac{240}{12} + \frac{120}{12} \\ &= \frac{360}{12} \\ &= 30\end{aligned}$$

c)
$$\begin{aligned}9(9-1)(9-2)(9-3)(9-4) &= 9(8)(7)(6)(5) \\ &= 15\,120\end{aligned}$$

d)
$$\begin{aligned}5^5 - (3^5 + 2^5) &= 3125 - (243 + 32) \\ &= 3125 - 275 \\ &= 2850\end{aligned}$$

e)
$$\begin{aligned}1 - \left(\frac{2}{3}\right)^4 &= 1 - \frac{16}{81} \\ &= \frac{81}{81} - \frac{16}{81} \\ &= \frac{65}{81}\end{aligned}$$

f)
$$\begin{aligned}\left(\frac{1}{2}\right)^2\left(\frac{1}{4}\right)^3 &= \frac{1}{4}\left(\frac{1}{64}\right) \\ &= \frac{1}{256}\end{aligned}$$

Chapter 2 Prerequisite Skills Question 7 Page 62

a) For n = 6,

$$\begin{aligned} n(n-1)(n-2)(n-3) &= 6(6-1)(6-2)(6-3) \\ &= 6(5)(4)(3) \\ &= 360 \end{aligned}$$

b) For a = 10,

$$\begin{aligned} (a+3)(a+2)(a+1) &= (10+3)(10+2)(10+1) \\ &= (13)(12)(11) \\ &= 1716 \end{aligned}$$

c) For x = 7,

$$\begin{aligned} &\frac{x(x-1)(x-2)(x-3)(x-4)}{(x+1)(x+2)} \\ &= \frac{7(7-1)(7-2)(7-3)(7-4)}{(7+1)(7+2)} \\ &= \frac{7(6)(5)(4)(3)}{(8)(9)} \\ &= \frac{2520}{72} \\ &= 35 \end{aligned}$$

d) For m = 0.4, n = 0.6, r = 3, and q = 4,

$$\begin{aligned} n^r \times m^q &= 0.6^3 \times 0.4^4 \\ &= 0.216 \times 0.0256 \\ &= 0.005\,529\,6 \end{aligned}$$

Chapter 2 Prerequisite Skills Question 8 Page 62

a) For n = 5,

$$\begin{aligned} n(n-1)(n-2) &= 5(5-1)(5-2) \\ &= 5(4)(3) \\ &= 60 \end{aligned}$$

b) For n = 5 and m = 3,

$$\begin{aligned} (n+2)(m+6) &= (5+2)(3+6) \\ &= (7)(9) \\ &= 63 \end{aligned}$$

c) For n = 5 and m = 3,

$$\begin{aligned} \frac{n(n-1)(n-2)}{m+1} &= \frac{5(5-1)(5-2)}{3+1} \\ &= \frac{5(4)(3)}{4} \\ &= \frac{60}{4} \\ &= 15 \end{aligned}$$

d) For n = 5 and m = 3,

$$\begin{aligned} &(n+5)(n+4)(n+3)+(m-1)(m-2) \\ &= (5+5)(5+4)(5+3)+(3-1)(3-2) \\ &= (10)(9)(8)+(2)(1) \\ &= 720+2 \\ &= 722 \end{aligned}$$

Chapter 2 Prerequisite Skills Question 9 Page 63

a)

$$\begin{aligned} x(x-1)(x-2) &= (x^2-x)(x-2) \\ &= x^3-2x^2-x^2+2x \\ &= x^3-3x^2+2x \end{aligned}$$

b)

$$\begin{aligned} &(x+1)(x+2)+(x-1)(x-2) \\ &= x^2+2x+x+2+x^2-2x-x+2 \\ &= 2x^2+4 \end{aligned}$$

c) $\dfrac{(x+5)(x+4)}{x+4} = \dfrac{(x+5)\cancel{(x+4)}}{\cancel{x+4}}$
$= x+5$

d) $\dfrac{x(x-1)(x-2)(x-3)}{x(x-1)} = \dfrac{\cancel{x}\cancel{(x-1)}(x-2)(x-3)}{\cancel{x}\cancel{(x-1)}}$
$= (x-2)(x-3)$
$= x^2 - 3x - 2x + 6$
$= x^2 - 5x + 6$

Chapter 2 Prerequisite Skills Question 10 Page 63

a) $\dfrac{8\times7\times6\times5\times4\times3\times2\times1}{5\times4\times3\times2\times1}$
$= \dfrac{40\,320}{120}$
$= 336$

$\dfrac{8\times7\times6\times5\times4\times3\times2\times1}{5\times4\times3\times2\times1}$
$= \dfrac{8\times7\times6\times\cancel{5\times4\times3\times2\times1}}{\cancel{5\times4\times3\times2\times1}}$
$= 336$

b) $\dfrac{8\times7\times6\times5\times4\times3\times2\times1}{2\times2\times2}$
$= \dfrac{40\,320}{8}$
$= 5040$

$\dfrac{8\times7\times6\times5\times4\times3\times2\times1}{2\times2\times2}$
$= \dfrac{8\times7\times6\times5\times\cancel{4}\times3\times\cancel{2}\times1}{\cancel{2\times2}\times\cancel{2}}$
$= 5040$

c) $\dfrac{5\times4\times3\times2\times1}{5\times4} = \dfrac{120}{20}$
$= 6$

$\dfrac{5\times4\times3\times2\times1}{5\times4} = \dfrac{\cancel{5\times4}\times3\times2\times1}{\cancel{5\times4}}$
$= 6$

d) $\dfrac{11\times10\times9\times8\times7\times6}{5\times4\times3\times2\times1}$
$= \dfrac{332\,640}{120}$
$= 2772$

$\dfrac{11\times10\times9\times8\times7\times6}{5\times4\times3\times2\times1}$
$= \dfrac{11\times\overset{2}{\cancel{10}}\times9\times\overset{2}{\cancel{8}}\times7\times\cancel{6}}{\cancel{5}\times\cancel{4}\times\cancel{3\times2}\times1}$
$= 2772$

Chapter 2 Prerequisite Skills Question 11 Page 63

a) The probability of rolling a 4 on a single die is $\dfrac{1}{6}$.

b) From a table of all possible sums, the probability of rolling a sum of 6 on a pair of dice is $\dfrac{5}{36}$.

First Die / Second Die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

c) The probability of flipping tails with a coin is $\frac{1}{2}$.

d) From a table of all possible combinations , the probability of flipping two heads with two coins is $\frac{1}{4}$.

First Coin / Second Coin	H	T
H	H H	T H
T	H T	T T

e) The probability of selecting a blue ball from a bag containing a red, a blue, a green, a yellow, a brown, and a purple ball is $\frac{1}{6}$.

Chapter 2 Prerequisite Skills Question 12 Page 63

a) Since the outcome of flipping a coin does not affect the outcome of rolling a die, these are independent events.

b) Since the outcome of dealing a first card affects the second card dealt, these are dependent events.

c) Since the outcome of first die does not affect the outcome of the second die, these are independent events.

d) Since the outcome of randomly selecting a date from a calendar does not affect the outcome of randomly selecting someone's name from a list, these are independent events.

Chapter 2 Prerequisite Skills Question 13 Page 63

a) The probability of rolling a 3 on an icosahedron die is $\frac{1}{20}$.

b) The probability of rolling a 4 on an icosahedron die is $\frac{1}{20}$.

c) The probability of rolling a 3 or a 4 on an icosahedron die is $\frac{2}{20}$, or $\frac{1}{10}$.

d) The probability of rolling an even number (2, 4, 6, 8, 10, 12, 14, 16, 18, 20) on an icosahedron die is $\frac{10}{20}$, or $\frac{1}{2}$.

e) The probability of rolling a prime number (2, 3, 5, 7, 11, 13, 17, 19) on an icosahedron die is $\frac{8}{20}$, or $\frac{2}{5}$.

f) The probability of rolling a number greater than 6 (7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20) on an icosahedron die is $\frac{14}{20}$, or $\frac{7}{10}$.

Chapter 2 Prerequisite Skills Question 14 Page 63

a) The event of rolling a 3 with a single die and the event of rolling an even number on a single die are mutually exclusive.

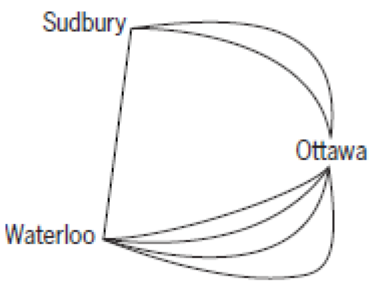
b) The event of randomly selecting a student with blue eyes and the event of randomly selecting a student with brown hair are non-mutually exclusive.

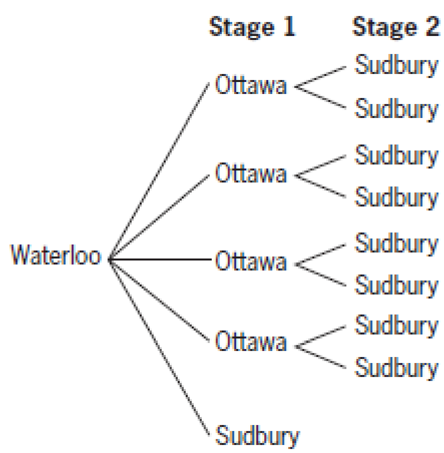
c) The event of selecting a face card from a deck and the event of selecting a numbered card from a deck are mutually exclusive.

d) The event of selecting a red sweater and the event of selecting a wool sweater are non-mutually exclusive.

e) The event of randomly selecting a vowel from the alphabet and the event of randomly selecting "A" or "E" from the alphabet are non-mutually exclusive.

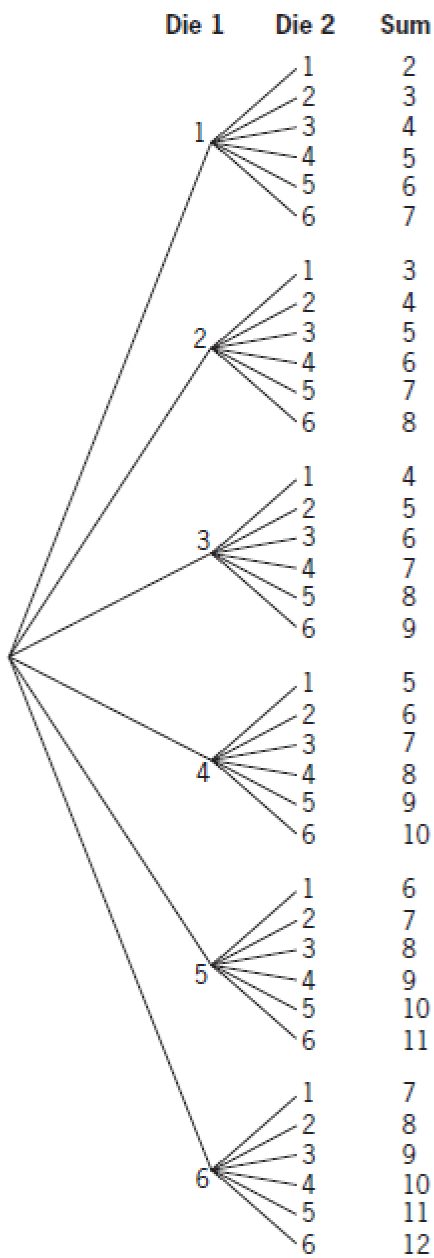
Chapter 2 Prerequisite Skills Question 15 Page 63





Chapter 2 Prerequisite Skills Question 16 Page 63

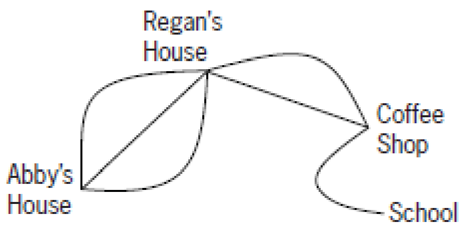
Second Die \ First Die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

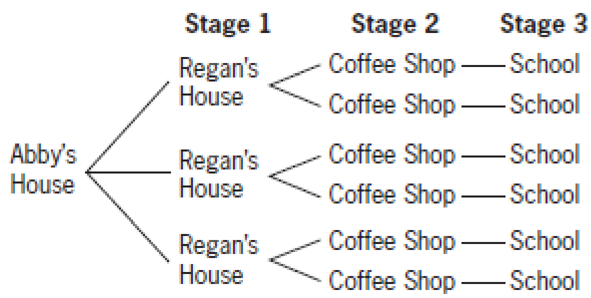


Chapter 2 Section 1 Organized Counting

Chapter 2 Section 1 Example 1 Your Turn Page 65

a)

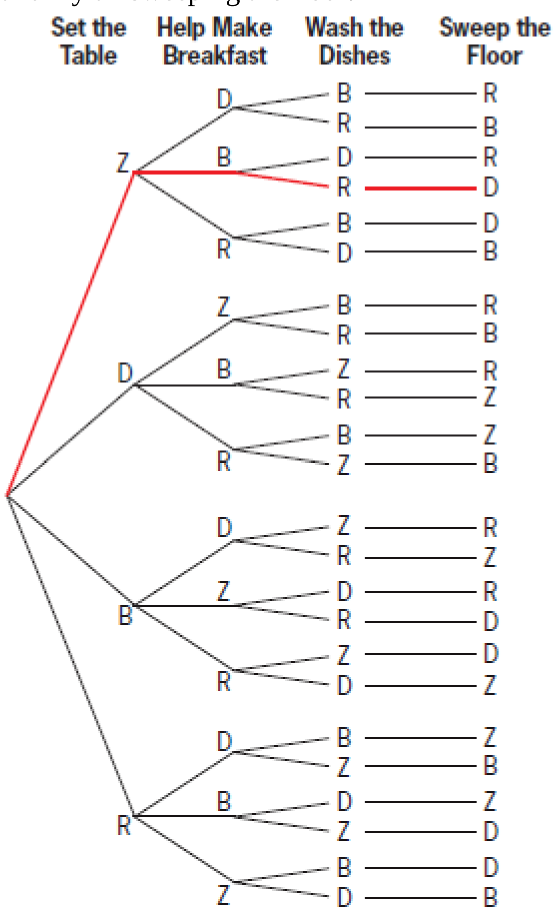




b) There are six branches in the final stage. So, there are six different routes that Abby could take.

Chapter 2 Section 1 Example 2 Your Turn Page 67

a) and **b)**
The path illustrates Zach setting the table, Ben helping make breakfast, Rhys washing the dishes, and Dylan sweeping the floor.



c) There are 24 branches in the final stage of the tree diagram. So, there are 24 different arrangements for doing the chores.

Chapter 2 Section 1 R1 Page 67

Answers may vary. While a tree diagram does show all possible outcomes, the actual drawing gets more complicated with more stages. For three stages, a tree diagram is an efficient way to illustrate the outcomes of three spins.

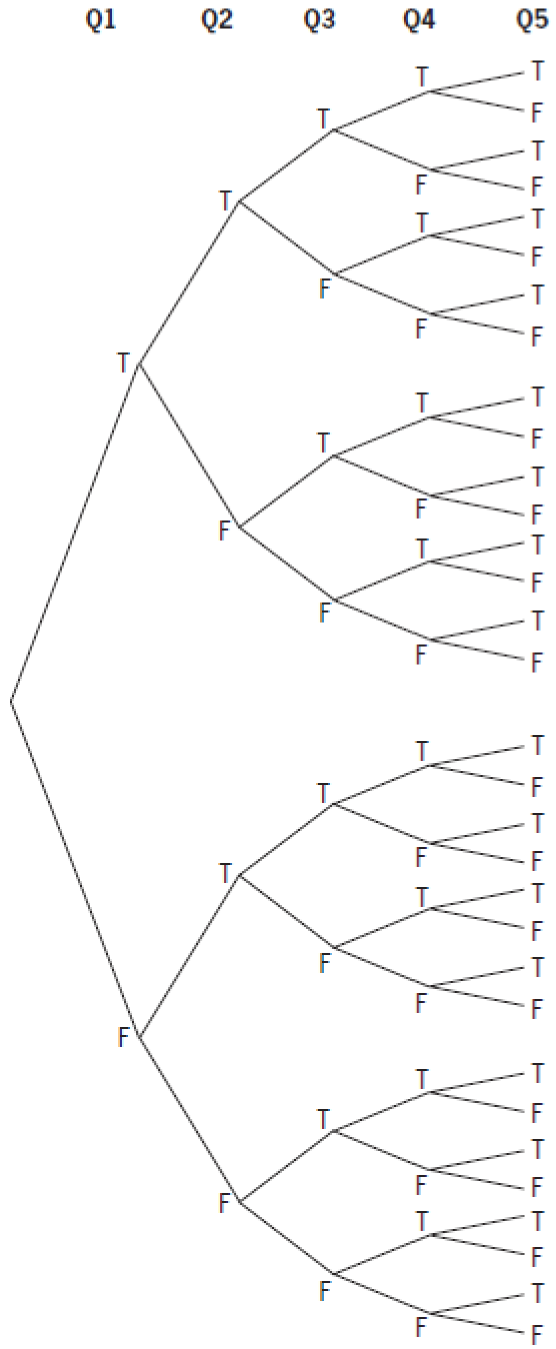
Chapter 2 Section 1 R2 Page 67

Answers may vary.
a) I prefer a table of values to show the outcomes of rolling two dice. It is faster to create and more efficient than a tree diagram.

b) This is a personal preference. A chart may be less efficient than a tree diagram when there are three or fewer stages.

Chapter 2 Section 1 Question 1 Page 67

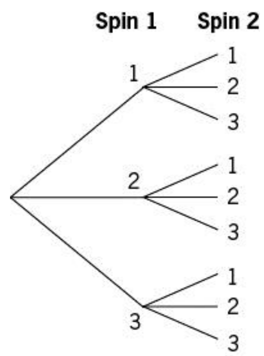
T, T, T, T, T	T, F, F, T, T	T, T, F, F, F	T, F, F, T, F
T, T, T, T, F	F, F, T, T, T	T, F, F, F, T	T, F, T, F, F
T, T, T, F, T	F, T, F, T, T	F, F, F, T, T	F, F, F, F, T
T, T, F, T, T	F, T, T, F, T	F, F, T, F, T	F, F, F, T, F
T, F, T, T, T	F, T, T, T, F	F, F, T, T, F	F, F, T, F, F
F, T, T, T, T	T, F, T, F, T	F, T, F, F, T	F, T, F, F, F
T, T, T, F, F	T, F, T, T, F	F, T, F, T, F	T, F, F, F, F
T, T, F, F, T	T, T, F, T, F	F, T, T, F, F	F, F, F, F, F



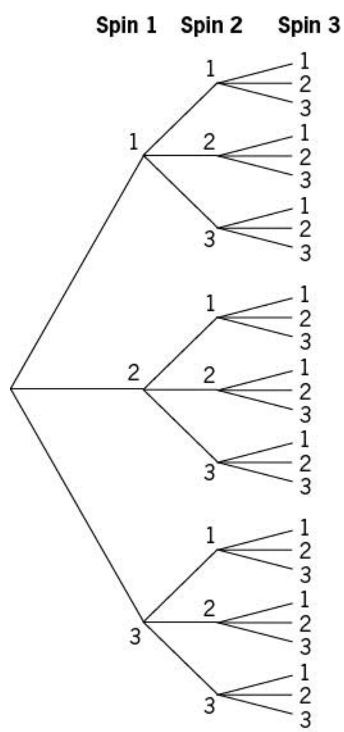
There are 32 branches in the final stage of the tree diagram. So, there are 32 different ways to answer the five questions.

Chapter 2 Section 1 Question 2 Page 67

a) There are 9 branches in the final stage of the tree diagram. So, there are 9 different outcomes if the spinner is spun twice.

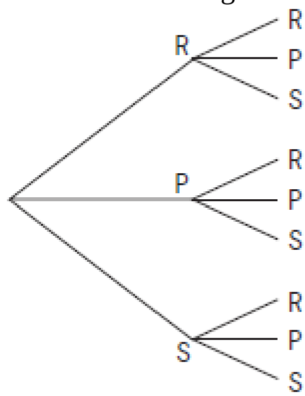


b) There are 27 branches in the final stage of the tree diagram. So, there are 27 different outcomes if the spinner is spun three times.



Chapter 2 Section 1 Question 3 Page 68

In the game of Rock-Paper-Scissors, there are a total of three choices (branches) in the first stage. Then in the second stage, there are three choices for each branch in the first stage for a total of nine outcomes. Diagram B illustrates this.

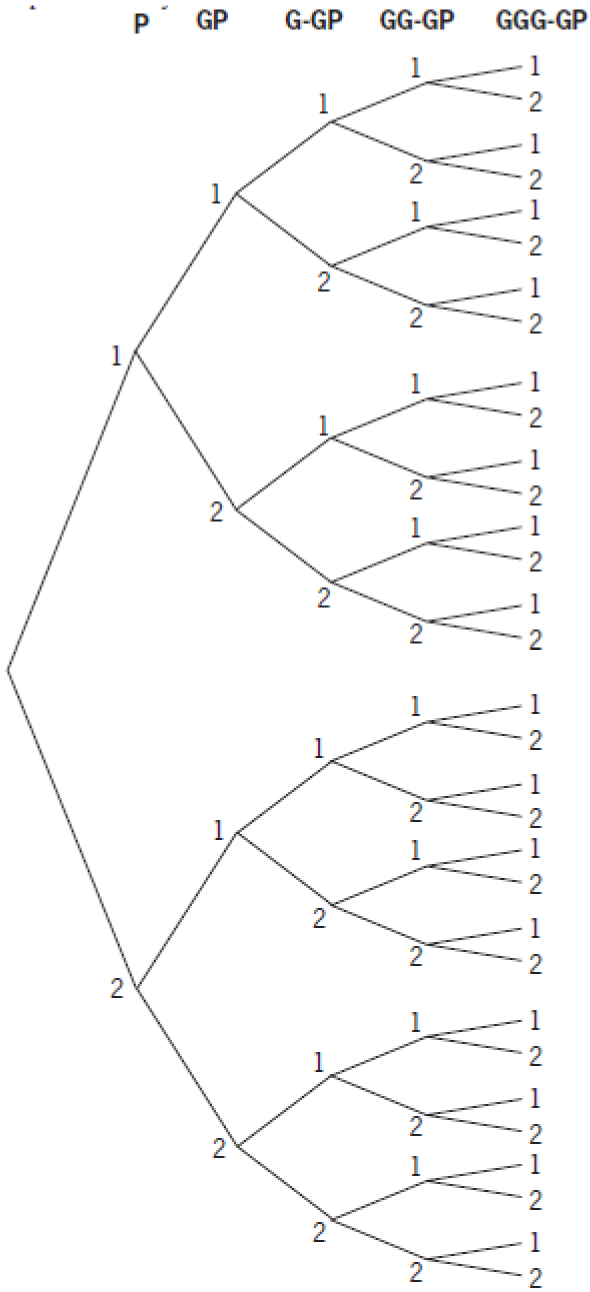


Chapter 2 Section 1 Question 4 Page 68

If a single die is rolled twice, the tree diagram will have two stages. The first stage has six branches, and the second stage has six branches for each branch in the first stage for a total of 36. The best choice is **C**: two stages, six branches per stage, 36 outcomes.

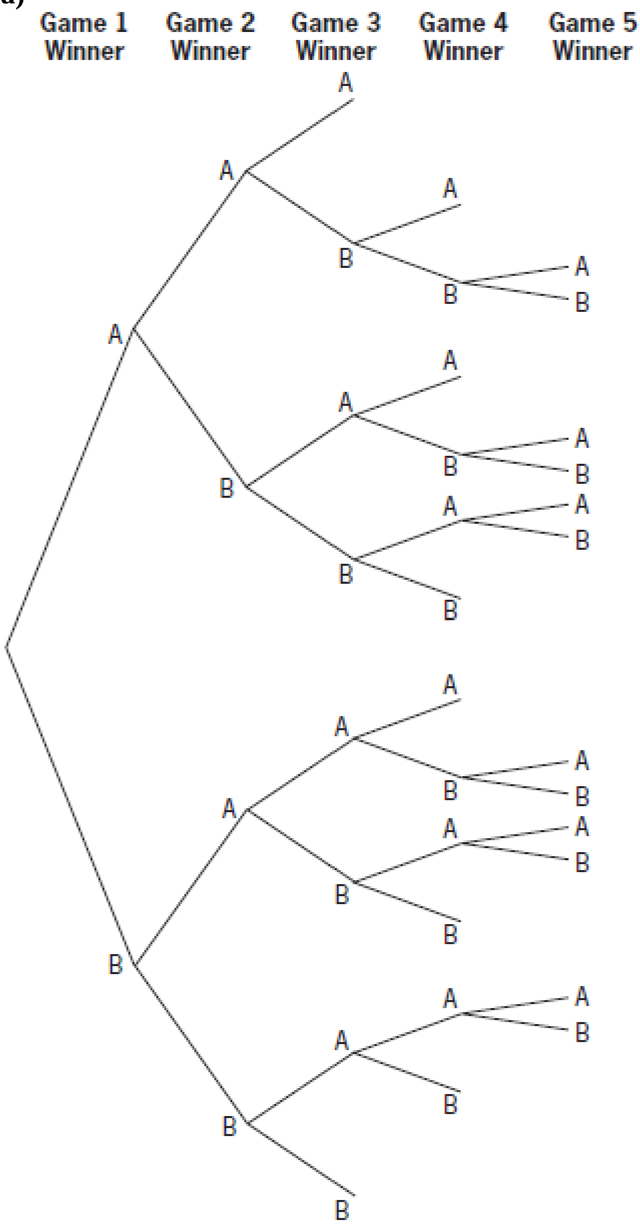
Chapter 2 Section 1 Question 5 Page 68

Answers may vary. Let each set of parents be represented by 1 and 2.


P
GP
G-GP
GG-GP
GGG-GP

Chapter 2 Section 1 **Question 6 Page 68**

a)



b) Use the tree diagram to write the list of outcomes.

AAA	ABAA	BAAA	BBAAA
AABA	ABABA	BAABA	BBAAB
AABBA	ABABB	BAABB	BBAB
AABBB	ABBAA	BABAA	BBB
	ABBAB	BABAB	
	ABBB	BABB	

c) There are 20 branch paths in the tree diagram. So, there are 18 different outcomes possible.

Chapter 2 Section 1 Question 7 Page 68

a)

Member 1	Member 2
A	B or C or D or E or F
B	A or C or D or E or F
C	A or B or D or E or F
D	A or B or C or E or F
E	A or B or C or D or F
F	A or B or C or D or E

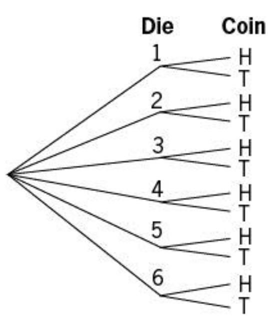
b) Once the first member of the committee is chosen, there are five possible choices for the second member. There are 6(5), or 30 outcomes possible.

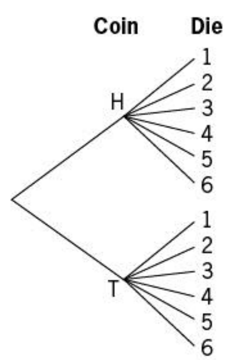
c) If order is not important, the chart changes. Now, there are only 15 outcomes possible.

Member 1	Member 2
A	B or C or D or E or F
B	C or D or E or F
C	D or E or F
D	E or F
E	F

Chapter 2 Section 1 Question 8 Page 68

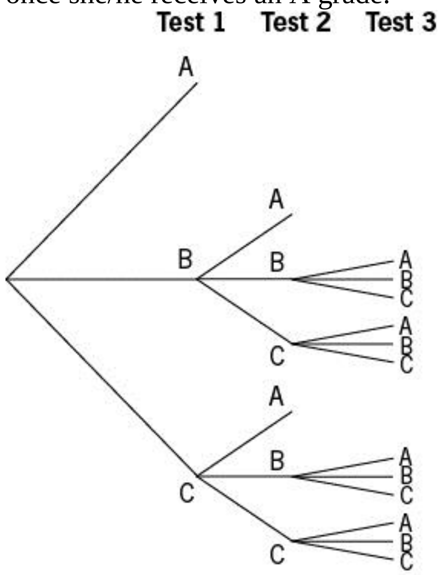
Since the die and coin results are independent events, it does matter whether the die is rolled first or the coin is flipped first. This is confirmed by tree diagrams with either event being first resulting in the same outcomes.





Chapter 2 Section 1 Question 9 Page 68

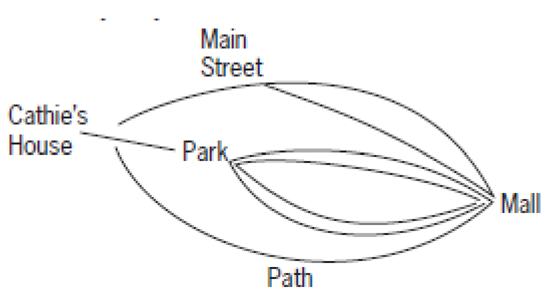
Answers may vary.
Assume that the possible test results are letter grades A, B, and C and that a student stops testing once she/he receives an A grade.

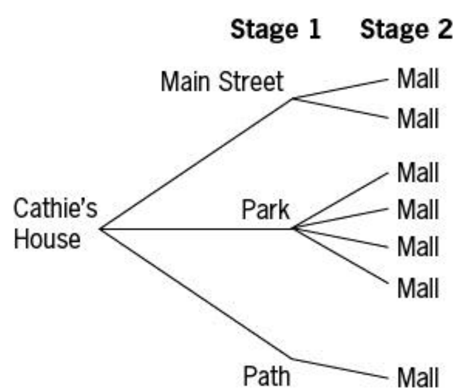


There are 15 different sets of results possible.

Chapter 2 Section 1 Question 10 Page 69

Answers may vary.
a)





b) There are seven different routes for Cathie to get to the shopping mall.

Chapter 2 Section 1 Question 11 Page 69

a) Answers may vary.

Card 1	Card 2
A	A or J or K or Q
J	A or J or K or Q
K	A or J or K or Q
Q	A or J or K or Q

b) Answers may vary.

Card 1	Card 2
A	J or K or Q
J	A or K or Q
K	A or J or Q
Q	A or J or K

c) The first scenario (with replacement/repetition allowed) results in 4(4), or 16 possible outcomes. The second scenario (without replacement/repetition not allowed) results in 4(3), or 12 possible outcomes. Without replacement, there is one less choice for card 2.

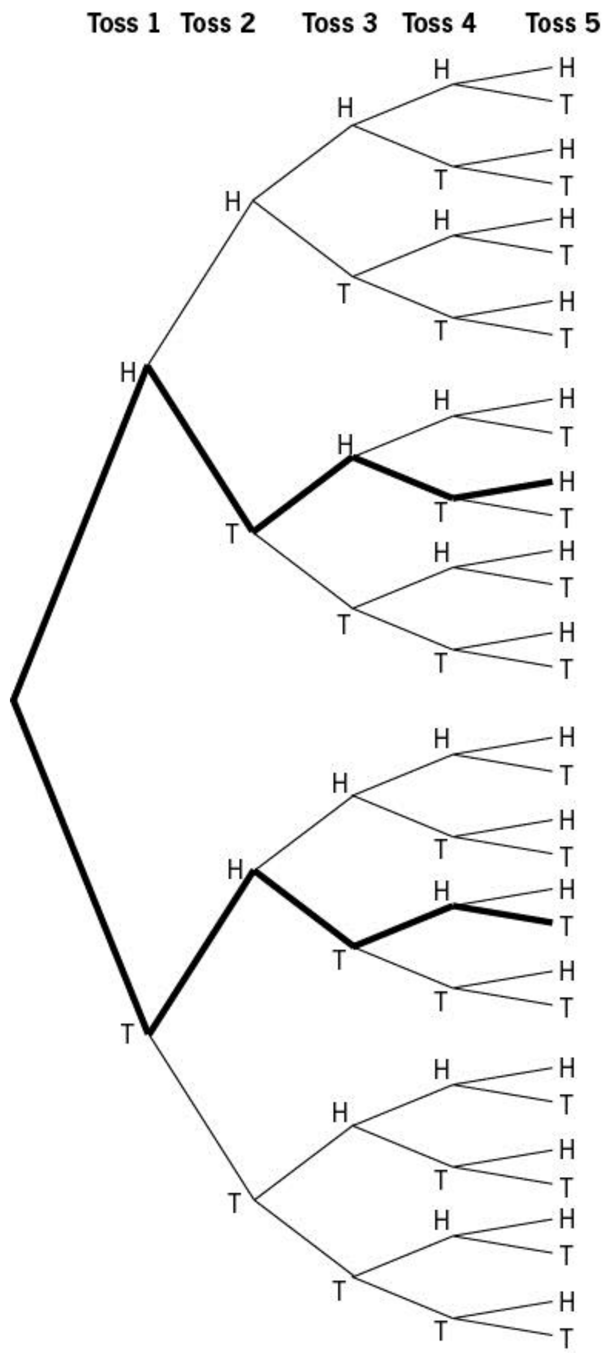
Chapter 2 Section 1 Question 12 Page 69

a) Since there are three colour choices for siding, stage 1 of a tree diagram will have 3 choices. Next, there are five colour choices for trim. So, stage 2 will have five branches for each siding branch, or a total of 15 options listed. Finally, there are three colour choices for the garage doors. So, stage 3 will have three branches for each of the 15 siding-trim branches, or a total of 45 outcomes.

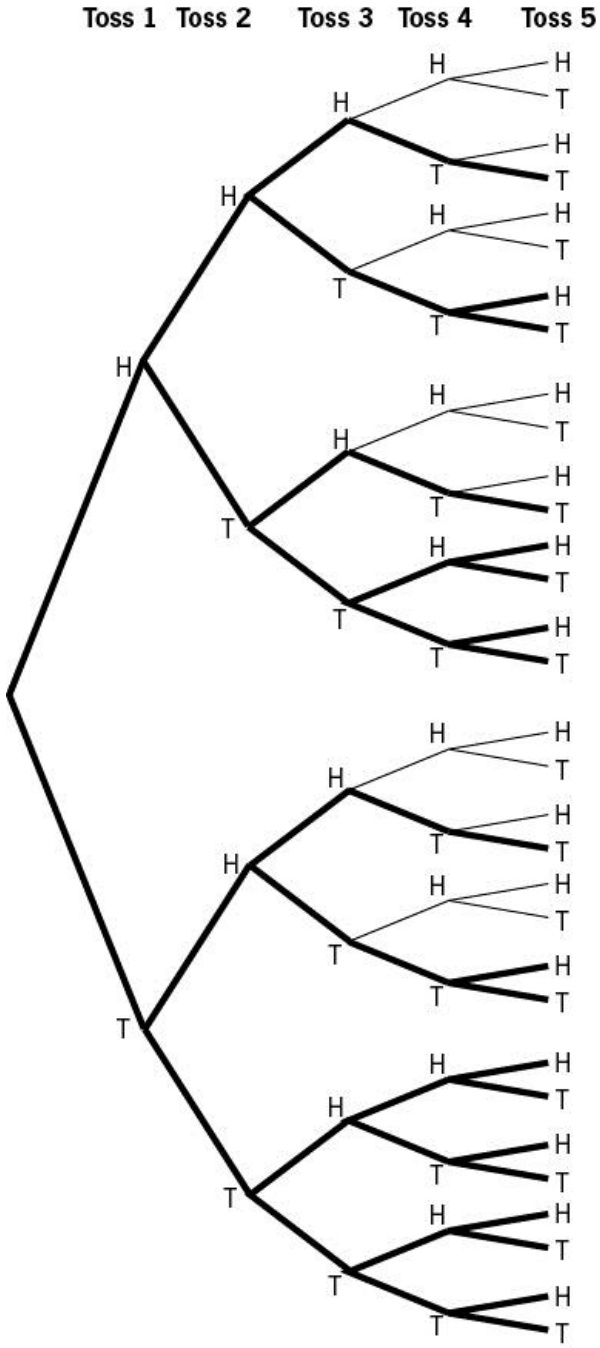
b) An additional siding colour results in 4(5)(3), or 60 different colour configurations. This is an increase of 15 choices. An additional trim colour results in 3(6)(3), or 54 different colour configurations. This is an increase of only 9 choices. So, increasing the number of siding colours would increase the number of choices by a greater amount.

Chapter 2 Section 1 Question 13 Page 69

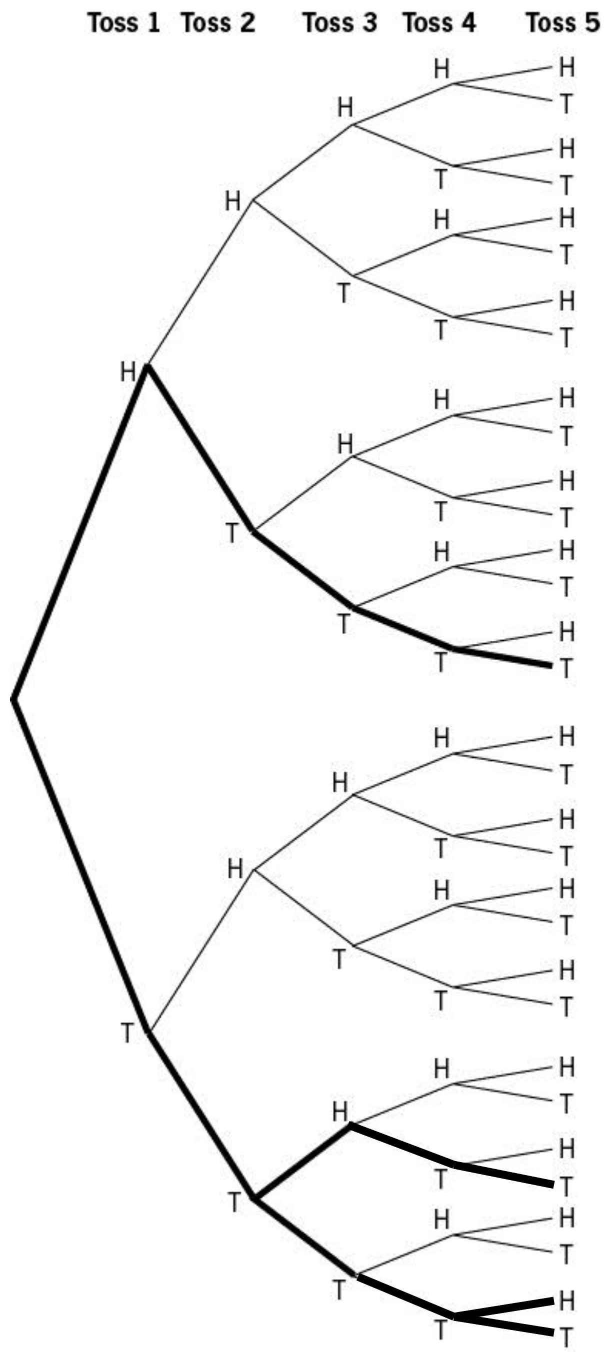
a) Make a tree diagram to show the results of flipping a coin five times. There are two possible results with no consecutive flips of heads or tails.



b) Make a tree diagram to show the results of flipping a coin five times. There are 19 possible results with at least two consecutive flips of tails.



c) Make a tree diagram to show the results of flipping a coin five times. There are 4 possible results with two flips of two consecutive tails.



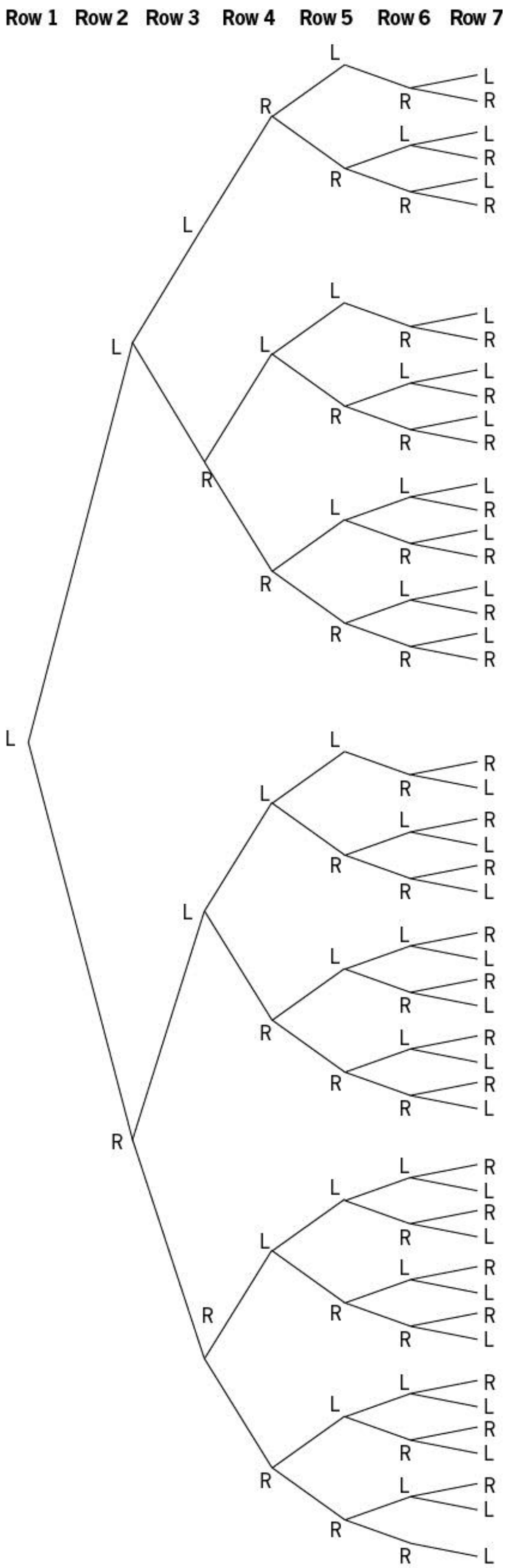
Chapter 2 Section 1 Question 14 Page 69

Since there are two choices for area code, stage 1 of a tree diagram will have 2 choices. Next, the 519-branch has 80 choices for prefix, and the 226-branch has 39 choices for prefix. So, stage 2 will have 80 + 39, or a total of 119 options listed. From each of these branches there are 10 choices (0 through 9) for the first of four digits, from each of these there are 10 choices (0 through 9) for the second of four digits, from each of these there are 10 choices (0 through 9) for the third of four digits, and finally from each of these there are 10 choices (0 through 9) for the last of four digits. So, the final stage will have 119(10)(10)(10)(10), or 1 190 000 different local phone numbers Sarah can call.

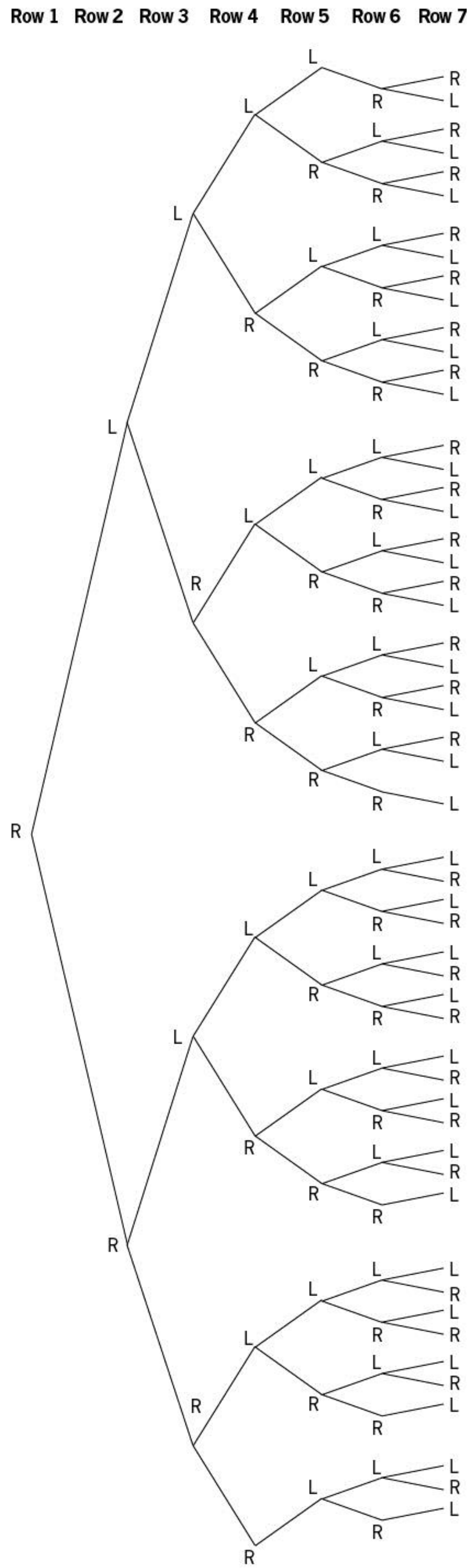
Chapter 2 Section 1 Question 15 Page 69

Assume rows on checkerboard are numbered 0 to 7. The portion of the tree diagram that starts with a move diagonally left has 49 possible paths to the opposite side. Similarly, the portion of the tree diagram that starts with a move diagonally right has 54 possible paths to the opposite side. The total number of possible paths to the opposite side is 49 + 54, or 103.

Move Diagonally Left

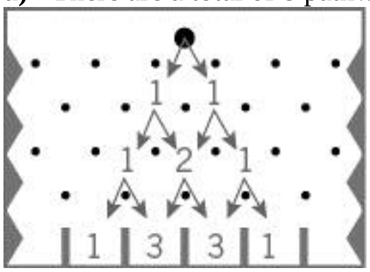


Move Diagonally Right



Chapter 2 Section 1 Question 16 Page 69

a) There are a total of 8 pathways to the bottom of the board.



b) Assume that a level is represented by a row of pegs. Then, five levels would provide 1 + 4 + 6 + 4 + 1, or 16 pathways to the bottom of the board.

c) Assume that a level is represented by a row of pegs. Look for a pattern that relates the row number to the number of pathways.

Row Number	Number of Pathways
1	2
2	4
3	8
4	16
⋮	⋮
n	2^n

If the board were extended to n levels, there would be 2^n pathways to the bottom of the board.

Chapter 2 Section 2 The Fundamental Counting Principle

Chapter 2 Section 2 Example 1 Your Turn Page 71

Using the fundamental counting principle, there are 3 × 2 × 10, or 60 different configurations of the smartphone available.

Chapter 2 Section 2 Example 2 Your Turn Page 72

a) For a six-letter password with repetition allowed, there are 26 × 26 × 26 × 26 × 26 × 26, or 308 915 776 different possibilities.

b) For a six-letter password with repetition allowed and the letters can be capitals or lower case, there are 52 × 52 × 52 × 52 × 52 × 52, or 19 770 609 664 different possibilities.

Chapter 2 Section 2 Example 3 Your Turn Page 73

From a class of 25 students, three of them are selected to attend a workshop in 25 × 24 × 23, or 13 800 ways.

Chapter 2 Section 2 R1 Page 73

Johnny is wrong. He should apply the fundamental counting principle, then there are 4 × 8 × 3, or 96 choices in total.

Chapter 2 Section 2 R2 Page 73

Answers may vary.
The fundamental counting principle is the product of the number of ways multiple events can occur. For example, there are 3 flavours of ice cream and 6 choices of toppings to create a sundae. Event one, choosing ice cream flavour, can happen in 3 ways. Event two, choosing a topping, can happen in 6 ways. The result is 3 × 6, or 18 different 1-topping ice cream sundaes.

Chapter 2 Section 2 Question 1 Page 73

a) When a coin is tossed twice, there are 2(2), or 4 possible outcomes.

b) When a coin is tossed three times, there are 2(2)(2), or 8 possible outcomes.

c) When a coin is tossed four times, there are 2(2)(2)(2), or 16 possible outcomes.

d) When a coin is tossed *n* times, there are 2^n possible outcomes.

Chapter 2 Section 2 Question 2 Page 73

a) From a committee of 15 people, there are 15(14), or 210 ways to choose a president and vice president.

b) From a committee of 15 people, there are 15(14)(13), or 2730 ways to choose a president, vice president, and secretary.

Chapter 2 Section 2 Question 3 Page 73

The total number of choices when selecting patio stones is 10(8)(3), or 240 choices.

Chapter 2 Section 2 Question 4 Page 73

a) There are 3 courses, or options.

b) The first course, appetizers, has 4 choices. The second course, main course, has 5 choices. The third course, dessert, has 3 choices.

c) The three-course menu in a restaurant provides 4(5)(3), or 60 meal choices for customers.

Chapter 2 Section 2 Question 5 Page 73

A computer can randomly select three different numbers from between 1 and 100 in 100 × 99 × 98 ways. Answer C.

Chapter 2 Section 2 Question 6 Page 74

A contestant can randomly select a letter of the alphabet from a spinner and roll a standard die in 26(6), or 156 ways. Answer B.

Chapter 2 Section 2 Question 7 Page 74

a) A two-digit number chosen from five digits with repetition can be formed in 5(5), or 25 ways.

b) A two-digit number chosen from five digits without repetition can be formed in 5(4), or 20 ways.

Chapter 2 Section 2 Question 8 Page 74

a) When rolling two 4-sided dice, there are 4(4), or 16 possible outcomes.

b) When rolling three 4-sided dice, there are 4(4)(4), or 64 possible outcomes.

c) When rolling two 8-sided dice, there are 8(8), or 64 possible outcomes.

d) When rolling four 8-sided dice, there are 8(8)(8)(8), or 4096 possible outcomes.

e) When rolling two 12-sided dice, there are 12(12), or 144 possible outcomes.

f) When rolling five 12-sided dice, there are 12(12)(12)(12)(12), or 248 832 possible outcomes.

g) When rolling k n-sided dice, there are n^k possible outcomes.

Chapter 2 Section 2 Question 9 Page 74

The business card design software has 25(38)(20), or 19 000 card design available to the user.

Chapter 2 Section 2 Question 10 Page 74

The customer has 5(6)(3), or 90 gift wrapping choices.

Chapter 2 Section 2 Question 11 Page 74

When rolling five dice once, there are 6^5, or 7776 possible outcomes.

Chapter 2 Section 2 Question 12 Page 74

a) If repetition of times during each hour is permitted, there are 60^4, or 12 960 000 different arrangements of winning times.

b) If repetition of times during each hour is not permitted, there are 60(59)(58)(57), or 11 703 240 different arrangements of winning times.

Chapter 2 Section 2 Question 13 Page 74

a) If repetition is permitted, there are 60^3, or 216 000 unique three-digit lock combinations possible.

b) If repetition is not permitted, there are 60(59)(58), or 205 320 unique 3-digit lock combinations possible.

Chapter 2 Section 2 Question 14 Page 74

a) For an eight-character password containing digits and capital and lower-case letters with repetition, there are 62^8, or 218 340 105 584 896 choices available.

b) For an eight-character password beginning with four different capital letters and followed by four different digits, there are 26(25)(24)(23)(10)(9)(8)(7), or 1 808 352 000 choices available.

c) For an eight-character password containing one digit and seven letters with repetition, there are 10(8)(52)(52)(52)(52)(52)(52)(52), or 82 245 736 202 240 choices available.

Chapter 2 Section 2 Question 15 Page 74

a) For an Ontario licence plate with four letters followed by three digits, there are 26(26)(26)(26)(10)(10)(10), or 456 976 000 choices available.

b) For a Québec licence plate with three letters followed by three digits, there are 26(26)(26)(10)(10)(10), or 17 576 000 choices available.

c) For a Northwest Territories licence plate with six digits, there are 10^6, or 1 000 000 choices available.

Chapter 2 Section 2 Question 16 Page 75

An Alberta licence plate with three letters followed by 4 digits will have much fewer choices than an Ontario licence plate with four letters followed by three digits. There are 26 choices for a letter, while there are only 10 choices for a digit.

Chapter 2 Section 2 Question 17 Page 75

Angus can travel 4(6), or 24 ways from Halifax to Vancouver via Toronto.

Chapter 2 Section 2 Question 18 Page 75

The number of outcomes for all three events will be the same, 6^3, or 216.
In event one and two, the colour of the dice does not affect the choices for a die, and rolling three dice once has the same results as rolling one die three times.

Chapter 2 Section 2 Question 19 Page 75

Answers may vary.
a) My security code is 325. I pressed ENTER 145 times before I saw my code.
The actual number of possible outcomes for a three-digit security is 1000. Using a graphing calculator to randomly generate a three-digit code could possibly take more than 1000 presses of ENTER because duplicates occur or your code may never be generated.

b) It might take 100 times longer to break a five-digit code versus a three-digit code. The actual number of possible outcomes for a five-digit security is 100 000. Using a graphing calculator to randomly generate a five-digit code could possibly take more than 10 000 presses of ENTER because of duplicates or your code may never be generated.

Chapter 2 Section 2 Question 20 Page 75

Answers may vary.
To find the number of choices for each of the three toppings, factor 4080: $2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 17$. Using all the factors, create three values. For example, there could be $(2 \times 2 \times 2)$ choices for sauce, $(2 \times 3 \times 5)$ choices for actual topping ingredient, and 17 choices for cheese.
Another possibility is that this includes, say four different sizes (S, M, L, XL). Then, there are actually 1020 $(2 \times 2 \times 3 \times 5 \times 17)$ topping options.

Chapter 2 Section 2 Question 21 Page 75

a) A 10-question multiple-choice test, each with four possible answers, can be completed in 4^{10}, or 1 048 576 ways.

b) Since the student can leave answers blank, the total number of choices per question is now 5 and the 10 questions can be answered in 5^{10}, or 9 765 625 ways.

Chapter 2 Section 2 Question 22 Page 75

From ACMA-213 to ACMA-999, there are 786 plates.
From ACMB-000 to ACMZ-999, there are 25(10)(10)(10), or 25 000 plates.
From ACNA-000 to ACZZ-999, there are 13(26)(10)(10)(10), or 338 000 plates.
From ADAA-000 to AZZZ-999, there are 23(26)(26)(10)(10)(10), or 15 548 000 plates.
From BAAA-000 to CZZZ-999, there are 2(26)(26)(26)(10)(10)(10), or 35 152 000 plates.
From DAAA-000 to DAAZ-999, there are 26(10)(10)(10), or 26 000 plates.
From DABA-000 to DAZZ-999, there are 25(26)(10)(10)(10), or 650 000 plates.
From DBAA-000 to DMEK-999, there are 12(5)(11)(10)(10)(10), or 660 000 plates.
From DMEL-000 to DMEL-429, there are 429 plates.
There are 52 400 215 plates between ACMA-213 and DMEL-429.

Chapter 2 Section 2 Question 23 Page 75

a) There are three choices for the ones place. Since one digit is used, then there are five choices for the tens place. Similarly, there are then four choices for the hundreds place.
If repetition is not permitted, there are 4(5)(3), or 60 ways to form an even three-digit number from 1, 2, 3, 4, 5, and 6.

b) Since the hundreds place cannot contain 0, there are several cases to consider.
Case 0: No 0
There are two choices for the ones, four choices for the tens place, and three choices for the hundreds place: 24 ways

Case 1: 0 in the ones place
There are five choices for the tens place and four choices for the hundreds place: 20 ways.

Case 2: 0 in the tens place
There are two choices for the ones place and four choices for the hundreds place: 8 ways.

If repetition is not permitted, there are 24 + 20 + 8, or 52 ways to form an even three-digit number from 0, 1, 2, 3, 4, and 5.

Chapter 2 Section 2 Question 24 Page 75

A committee is being formed with one student from each grade, plus an additional student from either grade 11 or 12.
Case 1: one student from each grade
There are three choices for grade 9, five choices for grade 10, six choices for grade 11, and nine choices for grade 12: 3(5)(6)(9) or 810 ways.

Case 2: student from either grade 11 or 12
There are now five choices for grade 11 and eight choices for grade 12: 13 ways

The committee can be formed in 810 + 13, or 8223 ways.

Chapter 2 Section 2 Question 25 Page 75

For a string of five different letters that must begin with a vowel and end with a consonant,

First Letter (vowel)	Second Letter	Third Letter	Fourth Letter	Fifth Letter (consonant)
5 choices	24 choices	23 choices	22 choices	21 choices

There are 5(24)(23)(22)(21), or 1 275 120 ways to form the string.

Chapter 2 Section 3 Permutations and Factorials

Chapter 2 Section 3 Example 1 Your Turn Page 77

a) $$\begin{aligned}4! &= 4\times3\times2\times1\\ &= 24\end{aligned}$$

b) $$\begin{aligned}6! &= 6\times5\times4\times3\times2\times1\\ &= 720\end{aligned}$$

c) $$\begin{aligned}\frac{11!}{7!} &= \frac{11\times10\times9\times8\times\cancel{7\times6\times5\times4\times3\times2\times1}}{\cancel{7\times6\times5\times4\times3\times2\times1}}\\ &= 11\times10\times9\times8\\ &= 7920\end{aligned}$$

d) $$\begin{aligned}\frac{6!\times4!}{5!} &= \frac{6\times\cancel{5\times4\times3\times2\times1}\times4!}{\cancel{5\times4\times3\times2\times1}}\\ &= 6\times4\times3\times2\times1\\ &= 144\end{aligned}$$

Chapter 2 Section 3 Example 2 Your Turn Page 78

Since all eight 30-second advertisement time slots are to be arranged, use a factorial.

$$\begin{aligned} 8! &= 8\times7\times6\times5\times4\times3\times2\times1 \\ &= 40\,320 \end{aligned}$$

There are 40 320 ways that the eight advertisements can be assigned a time.

Chapter 2 Section 3 Example 3 Your Turn Page 78

There are three medals to be awarded. The medals can be awarded in 40(39)(38), or 59 280 ways.

Chapter 2 Section 3 Example 4 Your Turn Page 79

Consider the two football photos as a single photo with the other four photos. These can be arranged in 5! ways. The two football photos can be arranged in 2! ways. Then, the photos can be arranged in 5! × 2!, or 240 ways.

Chapter 2 Section 3 R1 Page 80

Arranging *r* people from a group of *n* people with regard to order will have more possibilities. For example, ABC can be arranged in 6 ways: ABC, ACB, BAC, BCA, CAB, and CBA. Without regard for order these arrangements are the same.

Chapter 2 Section 3 R2 Page 80

Answers may vary.
Using a calculator, 0! has a value of 1.
Look at the formula for permutations. Suppose three people are awarded first, second, and third prize. So, $r = n = 3$ and the formula becomes $\frac{3!}{0!}$. In order for this to make sense, choose 0! to be defined as 1.

Chapter 2 Section 3 Question 1 Page 80

a) $$\begin{aligned} 9! &= 9\times8\times7\times6\times5\times4\times3\times2\times1 \\ &= 362\,880 \end{aligned}$$

b) $$\begin{aligned} \frac{12!}{5!} &= \frac{12\times11\times10\times9\times8\times7\times6\times\cancel{5\times4\times3\times2\times1}}{\cancel{5\times4\times3\times2\times1}} \\ &= 12\times11\times10\times9\times8\times7\times6 \\ &= 3\,991\,680 \end{aligned}$$

c) For ${}_7P_7$, n = 7 and r = 7.

$$\begin{aligned}{}_7P_7 &= \frac{7!}{(7-7)!}\\ &= \frac{7!}{0!}\\ &= \frac{7\times6\times5\times4\times3\times2\times1}{1}\\ &= 5040\end{aligned}$$

d) For ${}_8P_5$, n = 8 and r = 5.

$$\begin{aligned}{}_8P_5 &= \frac{8!}{(8-5)!}\\ &= \frac{8!}{3!}\\ &= \frac{8\times7\times6\times5\times4\times \cancel{3!}}{\cancel{3!}}\\ &= 8\times7\times6\times5\times4\\ &= 6720\end{aligned}$$

Chapter 2 Section 3 Question 2 Page 80

a) For ${}_6P_4$, n = 6 and r = 4.

$$\begin{aligned}{}_6P_4 &= \frac{6!}{(6-4)!}\\ &= \frac{6!}{2!}\end{aligned}$$

b) For ${}_{15}P_6$, n = 15 and r = 6.

$$\begin{aligned}{}_{15}P_6 &= \frac{15!}{(15-6)!}\\ &= \frac{15!}{9!}\end{aligned}$$

c) $7\times6\times5\times4\times3\times2\times1 = 7!$

d) $$\begin{aligned}8\times7\times6\times5 &= \frac{8\times7\times6\times5\times \cancel{4!}}{\cancel{4!}}\\ &= \frac{8!}{4!}\end{aligned}$$

e) $$\begin{aligned}&n(n-1)(n-2)(n-3)\\ &= \frac{n(n-1)(n-2)(n-3)\times \cancel{(n-4)!}}{\cancel{(n-4)!}}\\ &= \frac{n!}{(n-4)!}\end{aligned}$$

f) $(n+1)\times(n)\times(n-1)\times\ldots\times3\times2\times1 = (n+1)!$

Chapter 2 Section 3 Question 3 Page 80

a) The number of permutations of n items is ${}_nP_n = n!$. So, $6! = {}_6P_6$.

b) $$\begin{aligned}91\times90\times89\times88\times87\times86 &= \frac{91\times90\times89\times88\times87\times86\times \cancel{85!}}{\cancel{85!}}\\ &= \frac{91!}{(91-6)!}\\ &= {}_{91}P_6\end{aligned}$$

c) $$\begin{aligned}\frac{18!}{12!} &= \frac{18!}{(18-6)!}\\ &= {}_{18}P_6\end{aligned}$$

Chapter 2 Section 3 Question 4 Page 80

$$\frac{96!}{24!} = \frac{96!}{(96-72)!}$$
$$= {}_{96}P_{72}$$

Answer C.

Chapter 2 Section 3 Question 5 Page 80

For the permutations of five items from a list of nine items, $n = 9$ and $r = 5$.

$$\begin{aligned} {}_9P_5 &= \frac{9!}{(9-5)!} \\ &= \frac{9!}{4!} \\ &= \frac{9 \times 8 \times 7 \times 6 \times 5 \times \cancel{4!}}{\cancel{4!}} \\ &= 9 \times 8 \times 7 \times 6 \times 5 \\ &= 15\,120 \end{aligned}$$

Answer B.

Chapter 2 Section 3 Question 6 Page 80

There are three places to be awarded. The places can be awarded in 15(14)(13), or 2730 ways.

Chapter 2 Section 3 Question 7 Page 80

There are four positions to be elected from 18 members. So, $n = 18$ and $r = 4$.

$$\begin{aligned} {}_{18}P_4 &= \frac{18!}{(18-4)!} \\ &= \frac{18!}{14!} \\ &= \frac{18 \times 17 \times 16 \times 15 \times \cancel{14!}}{\cancel{14!}} \\ &= 18 \times 17 \times 16 \times 15 \\ &= 73\,440 \end{aligned}$$

The four positions can be elected in 73 440 ways.

Chapter 2 Section 3 Question 8 Page 81

There are nine spots to be assigned from 22 baseball players. So, $n = 22$ and $r = 9$.

$$
\begin{aligned}
{}_{22}P_9 &= \frac{22!}{(22-9)!}\\
&= \frac{22!}{13!}\\
&= \frac{22\times21\times20\times19\times18\times17\times16\times15\times14\times \cancel{13!}}{\cancel{13!}}\\
&= 22\times21\times20\times19\times18\times17\times16\times15\times14\\
&= 180\,503\,769\,600
\end{aligned}
$$

The nine spots can be assigned in 180 503 769 600 ways.

Chapter 2 Section 3 Question 9 Page 81

a) $10\times9\times8\times7! = 10!$

b) $99\times98\times97! = 99!$

c) $90\times8! = 10\times9\times8!$
$= 10!$

d) $n(n-1)! = n!$

e) $(n+2)(n+1)n! = (n+2)!$

Chapter 2 Section 3 Question 10 Page 81

a) Since the salesperson must visit all 15 offices, use a factorial.

$$
\begin{aligned}
15! &= 15\times14\times13\times12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1\\
&= 1\,307\,674\,368\,000
\end{aligned}
$$

There are 1 307 674 368 000 ways that the salesperson can visit all 15 offices.

b) To visit 4 out of 15 offices on one day, n = 15 and r = 4.

$$
\begin{aligned}
{}_{15}P_4 &= \frac{15!}{(15-4)!}\\
&= \frac{15!}{11!}\\
&= \frac{15\times14\times13\times12\times \cancel{11!}}{\cancel{11!}}\\
&= 15\times14\times13\times12\\
&= 32\,760
\end{aligned}
$$

There are 32 760 ways that the salesperson can visit 4 of the 15 offices in one day.

c) To visit 3 out of 15 offices on one day, n = 15 and r = 3.

$$
\begin{aligned}
{}_{15}P_3 &= \frac{15!}{(15-3)!}\\
&= \frac{15!}{12!}\\
&= \frac{15\times14\times13\times \cancel{12!}}{\cancel{12!}}\\
&= 15\times14\times13\\
&= 2730
\end{aligned}
$$

There are 2730 ways that the salesperson can visit 3 of the 15 offices in one day. Then, there are 5(2730), or 13 650 ways to visit three different offices each day from Monday to Friday.

Chapter 2 Section 3 Question 11 Page 81

a) Since this is a 10-digit number with no repeats, use a factorial.

$$\begin{aligned}10! &= 10\times9\times8\times7\times6\times5\times4\times3\times2\times1\\ &= 3\,628\,800\end{aligned}$$

There are 3 628 800 10-digit numbers.

b) For a 7-digit from 10 digits with no repeats, n = 10 and r = 7.

$$\begin{aligned}{}_{10}P_7 &= \frac{10!}{(10-7)!}\\ &= \frac{10!}{3!}\\ &= \frac{10\times9\times8\times7\times6\times5\times4\times \cancel{3!}}{\cancel{3!}}\\ &= 10\times9\times8\times7\times6\times5\times4\\ &= 604\,800\end{aligned}$$

There are 604 800 7-digit numbers.

Chapter 2 Section 3 Question 12 Page 81

Assume repeats within the remaining choices are allowed and all other digits are different than 0. Consider the 00 as a single digit, so there are 7 digits in the password.

$7 \times 9^6 = 3\ 720\ 087$

The digits can be arranged in 3 720 087 ways.

Chapter 2 Section 3 Question 13 Page 81

a) Since all six members of the student council executive are to be in the photo, use a factorial.

$$\begin{aligned}6! &= 6\times5\times4\times3\times2\times1\\ &= 720\end{aligned}$$

There are 720 ways that the student council executive can line up.

b) Consider the president and vice president as a single person with the other 4 people. These can be arranged in 5! ways. The president and vice president can be arranged in 2! ways. Then, the executive can be arranged in 5! × 2!, or 240 ways.

c) There are 2! ways to arrange the president and vice president in the middle of the row. There are 4! ways to arrange the remaining members of the executive. So, there are 4! × 2!, or 48 ways to arrange the executive with the president and vice president in the middle.

Chapter 2 Section 3 Question 14 Page 81

Use a chart to organize the choices.

Seat Number												
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
7 choices	6 choices	6 choices	5 choices	5 choices	4 choices	4 choices	3 choices	3 choices	2 choices	2 choices	1 choice	1 choice

So, the six boys and seven girls can be arranged in 7! × 6!, or 3 628 800 ways so that none of the girls sit together.

Chapter 2 Section 3 Question 15 Page 81

The groups are in a given order and only the skaters are arranged within their group. So, the number of skating orders in each group is 5!. The number of orders for 4 groups is $(5!)^4$, or 207 360 000.

Chapter 2 Section 3 Question 16 Page 81

a) For ${}_nP_2 = 110$ and $r = 2$,

$$\begin{aligned}
{}_nP_r &= \frac{n!}{(n-r)!} \\
110 &= \frac{n!}{(n-2)!} \\
110 &= \frac{n\times(n-1)\times\cancel{(n-2)!}}{\cancel{(n-2)!}} \\
110 &= n\times(n-1) \\
110 &= n^2-n \\
0 &= n^2-n-110 \\
0 &= (n-11)(n+10)
\end{aligned}$$

$n - 11 = 0$ or $n + 10 = 0$

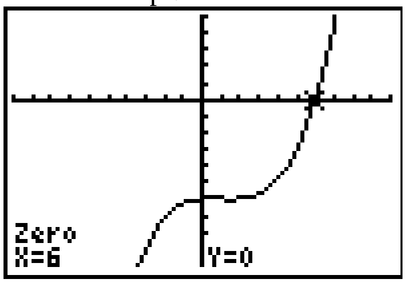
$n = 11$ $\qquad$ $n = -10$

Since n cannot be negative, the solution is $n = 11$.

b) For $P(n, 3) = 5!$ and $r = 3$,

$$\begin{aligned}
{}_nP_r &= \frac{n!}{(n-r)!} \\
5! &= \frac{n!}{(n-3)!} \\
120 &= \frac{n\times(n-1)\times(n-2)\times\cancel{(n-3)!}}{\cancel{(n-3)!}} \\
120 &= n\times(n-1)\times(n-2) \\
120 &= n^3-3n^2+2n \\
0 &= n^3-3n^2+2n-120
\end{aligned}$$

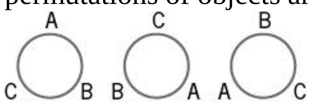
Use a graphing calculator to graph the corresponding function $y = x^3 - 3x^2 + 2x - 120$ and locate the x-intercept.



The solution is $n = 6$.

Chapter 2 Section 3 Question 17 Page 81

For each linear permutation, you can convert that arrangement into a circle arrangement by connecting the two ends. Instead of $n!$ arrangements, there are $(n - 1)!$ because circular permutations of objects are equivalent since the circle can be rotated.



Consider each couple as a single person. These can be arranged in $(10 - 1)!$ ways. Then, each couple can be arranged in 2! ways. So, the 10 couples can be arranged in $9! \times 2! \times 2! \times 2! \times 2! \times 2! \times 2! \times 2! \times 2! \times 2! \times 2!$, or 371 589 120 ways.

Chapter 2 Section 3 Question 18 Page 81

From the solution to question 17, there are $(n - 1)!$ circular permutations of objects. Choose a seat at the Round Table for King Arthur. This can be done 1 way. Then, the 23 knights can be seated in 23! ways. So, there are 1(23!), or 25 852 016 738 884 976 640 000 ways that King Arthur and the knights be seated at the Round Table.

Chapter 2 Section 3 Question 19 Page 81

a)
$$\begin{aligned} 9!! &= 1\times 3\times 5\times 7\times 9 \\ &= \frac{1\times 2\times 3\times 4\times 5\times 6\times 7\times 8\times 9}{2\times 4\times 6\times 8} \\ &= \frac{9!}{8!!} \end{aligned}$$

b) Using the pattern from part a), $n!! = \dfrac{n!}{(n-1)!!}$.

$$\begin{aligned} (2k+1)!! &= \frac{(2k+1)!}{((2k+1)-1)!!} \\ &= \frac{(2k+1)!}{2k!!} \end{aligned}$$

c) $(2n)!! = (2n)(2n-2)(2n-4)\ldots2$

$$= 2(n)[2(n-1)2(n-2)\ldots2]$$

$$= 2^n n!$$

Chapter 2 Section 3 Question 20 Page 81

Determine how many times 10 is a factor of the expansion of 30!. In particular, which numbers are multiples of 5 or 2.

$30\times29\times28\times27\times26\times25\times24\times23\times22\times21\times20\times19\times18\times17\times16\times15\times14\times13\times12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1$

A scan of the expansion list shows that there are many numbers that are multiples of 2 (even numbers), so concentrate on those that are multiples of 5.
There are six multiples of 5: 5, 10, 15, 20, 25, and 30.
So, 30! has six zeros at the end.

Chapter 2 Section 4 The Rule of Sum

Chapter 2 Section 4 Example 1 Your Turn Page 83

a) Use the rule of sum because either seven or eight or nine countries' flags will be flown.
7! + 8! + 9! = 408 240
The flags could be arranged in 408 240 ways.

b) If the host country's flag is always on the far left, there is only one choice for this position and one less flag to arrange for the other positions.
1(6!) + 1(7!) + 1(8!) = 46 080
The flags could be arranged in 46 080 ways.

Chapter 2 Section 4 Example 2 Your Turn Page 84

a) There are ${}_{13}P_3$ ways to select three hearts.

b) There are ${}_{4}P_3$ ways to select three aces.

c) The events "hearts" and "aces" are not mutually exclusive, since there is one ace of hearts, which has been counted twice.
Apply the principle of inclusion and exclusion.

Number of ways to get 3 aces or 3 hearts $= {}_{13}P_3 + {}_{4}P_3 - 1$

$$= \frac{13!}{10!} + \frac{4!}{1!} - 1$$

$$= 1716 + 24 - 1$$

$$= 1739$$

There are 1739 ways to get three aces or three hearts.

Chapter 2 Section 4 Example 3 Your Turn Page 85

To use the indirect method, first calculate the total number of outcomes without restrictions and the number of outcomes when the vowels must be together.
There are six letters in the word FACTOR. So, the total number of outcomes is 6!, or 720.

Treat the two vowels as a single letter, making 5 letters. Then, arrange the two of them among themselves.
5! × 2! = 240

Total – together = 6! – 5! × 2!
= 720 – 240
= 480
The vowels can be apart in 480 ways.

Chapter 2 Section 4 R1 Page 86

The indirect method is useful in determining the number of possible outcomes in this scenario, because it is simpler to calculate the number of executives without any males or females than all the possibilities for at least one male and one female.

Chapter 2 Section 4 R2 Page 86

Use the fundamental counting principle when the events are independent. For example, rolling a die twice. The outcome of the first event does not affect the second.
Use the rule of sum when events are mutually exclusive. For example, rolling a 1 or a 2. Both events cannot happen at the same time.

Chapter 2 Section 4 Question 1 Page 86

Use the rule of sum because either three or four toys will be picked from eight.

$$\begin{aligned} {}_8P_3 + {}_8P_4 &= \frac{8!}{5!} + \frac{8!}{4!} \\ &= 336 + 1680 \\ &= 2016 \end{aligned}$$

The three or four toys could be arranged in 2016 ways.

Chapter 2 Section 4 Question 2 Page 86

a) Recall the table of all possible outcomes for the sum of the dice when two standard dice are thrown. There are 6 ways to roll a sum of 7 and 2 ways to roll a sum of 11. So, there are 6 + 2, or 8 ways to roll a sum of 7 or 11 on two dice.

b) Recall the table of all possible outcomes for the sum of the dice when two standard dice are thrown. There are 6 ways to roll doubles and 12 ways to roll a sum divisible by 3. So, there are 6 + 12, or 18 ways to roll doubles or a sum divisible by three on two dice.

Chapter 2 Section 4 Question 3 Page 86

There are 6 ways to roll a die and 36 ways to roll two dice. So, there are 6 + 36, or 42 ways to roll either one or two dice. Answer A.

Chapter 2 Section 4 Question 4 Page 86

To use the indirect method, first calculate the total number of outcomes without restrictions and the number of outcomes when the even digits must be together.

There are five digits. So, the total number of outcomes is 5!, or 120.
Treat the two even digits as a single digit, making 4 digits. Then, arrange the two of them among themselves.
$4! \times 2! = 48$

Total – together = $5! - 4! \times 2!$
$= 120 - 48$
$= 72$
The even digits can be apart in 72 ways. Answer D.

Chapter 2 Section 4 Question 5 Page 86

Consider the cases for 1- to 5-digit numbers. For each case, there are 2 choices for the ones place in order to form an even number. Once a choice is made, the other place values can be determined.

Case 1: 1 digit
2 choices

Case 2: 2 digits
4(2), or 8 choices

Case 3: 3 digits
4(3)(2), or 24 choices

Case 4: 4 digits
4(3)(2)(2), or 48 choices

Case 5: 5 digits
4(3)(2)(1)(2), or 48 choices

So, there are 2 + 8 + 24 + 48 + 48, or 130 even numbers that can be formed.

b) To use the indirect method, consider the cases for numbers less than 3000.

Case 1: 4-digit even number starts with 1
1(3)(2)(2), or 12 choices

Case 2: 4-digit even number starts with 2
1(3)(2)(1), or 6 choices

The even numbers less than 3000 also include all the 1-, 2-, and 3-digit number found in part a).
Total – together = 130 – (12 + 6 + 2 + 8 + 24)
= 130 – 52
= 78
So, there are 78 even numbers greater than 3000.

Chapter 2 Section 4 Question 6 Page 86

Assume repeats are allowed. Use the rule of sum because the licence plate may have either two or three letters followed by four digits.
26(26)(10)(10)(10)(10) + 26(26)(26)(10)(10)(10)(10) = 182 520 000
The number of motorcycle plates that can be made is 182 520 000.

Chapter 2 Section 4 Question 7 Page 87

Use the rule of sum because the security code may have either five or six different letters.
To calculate the security codes in each event, use the fundamental counting principle or permutations.
$26(25)(24)(23)(22)(21) + {}_{26}P_5 = 173\ 659\ 200$
There are 173 659 200 distinct security codes possible.

Chapter 2 Section 4 Question 8 Page 87

a) Assume no repeats. Use the rule of sum because the child may have one, two, or three names. To calculate the number of possible names in each event, use the fundamental counting principle or permutations.
50 + 50(49) + 50(49)(48) = 120 100
There are 120 100 choices when naming their child.

b) Assume no repeats. Use the rule of sum, but now there 100 names.
100 + 100(99) + 100(99)(98) = 980 200
There are 980 200 choices when naming their child.

c) When the answers in parts a) and b) are expanded into factorial form, all three expressions in part b) are at least 2 times as big as those in part a). So, the result is more than $2^3 = 8$ times the answer in part a).

Chapter 2 Section 4 Question 9 Page 87

a) Since speaker P can go before speaker Q in several ways, different cases must be considered.

Case 1: 1 × 4! = 24

Case 2: 1 × 3 × 3! = 18

Case 3: 1 × 2 × 3! = 12

Case 4: 1 × 1 × 3! = 6

There are 24 + 18 + 12 + 6, or 60 ways that speaker P can go before speaker Q.

b) Answers may vary.
Question:
Five speakers, P, Q, R, S, and T, are available to address a meeting. The organizer must decide whether to have four or five speakers. How many options would the organizer have for the meeting?
Answer:
Use the rule of sum.
5! + 4! = 144
There are 144 options.

Chapter 2 Section 4 Question 10 Page 87

Since these events are not mutually exclusive, the principle of inclusion and exclusion needs to be considered.

To form a 5-digit number, the first digit cannot be 0, so there are 9 choices for the first digit.
9(10)(10)(10)(10) = 90 000
Then, calculate the number of 5-digit numbers that do not contain 4.
8(9)(9)(9)(9) = 52 488 (First digit cannot be 0 or 4.)
So, 90 000 – 52 488, or 37 512 5-digit numbers do contain 4.

Similarly, there are 37 512 5-digit numbers that do contain 6.

Next, numbers containing both 4 and 6 were counted twice, so subtract them once to compensate.
90 000 – 2(52 488) + 7(8)(8)(8)(8) = 13 696   (Total – [no 4] – [no 6] + [no 4 nor 6])

37 512 + 37 512 – 13 696 = 61328

Chapter 2 Section 4   Question 11   Page 87

Assume each name begins with a different letter.
Use the indirect method. First calculate the total number of ways to pull the names from the hat without restrictions and the number of ways to pull the names from the hat in alphabetical order.
There are 10!, or 3 628 800 ways to pull the names from the hat.
Then, there is only 1 way to pull the names from the hat in alphabetical order.

So, there are 3 628 800 – 1, or 3 628 799 ways the names could they be pulled from the hat so they are not in alphabetical order.

Chapter 2 Section 4   Question 12   Page 87

a) There are 6 ways for spin #1, 2 ways for spin #2, 2 ways for spin #3, and 2 ways for spin #4 to land on all the same colour. Use the rule of sum.
$6 \times 2 \times 2 \times 2 = 48$

There are 48 ways to land on the same colour on all four spins.

b) First, calculate how many ways the spinner could result in an even number. There are three sections that have an even number.
$3^4 = 81$

From part a) you know there are 48 ways to get the same colour on all four spins.

There are three ways to get all even and the same colour.

The number of ways to get all even or the same colour is 81 + 48 – 3, or 126.

Chapter 2 Section 4   Question 13   Page 87

Answers may vary. For each roll of the two dice, there are six ways to get doubles. Since rolling the dice once or twice or three times are mutually exclusive, there are 6 + 6 + 6, or 18 ways to get doubles in one or two or three rolls of the dice.

Chapter 2 Section 4   Question 14   Page 87

Morse code is used to represent 26 letters, 10 digits, and 8 punctuation symbols, or a total of 44 symbols. Since each character has two options (dot or dash), a maximum of six characters is needed: $2^6 = 64$.

Chapter 2 Section 4 Question 15 Page 87

a) There are 26 capital letters, 26 lower-case letters, and 10 digits, for a total of 62 options for each character. With no restrictions, there are $62^6 + 62^7 + 62^8$, or 221 918 520 426 688 ways to create a password with 6, 7, or 8 characters.

b) With no repetition allowed, there are ${}_{62}P_6 + {}_{62}P_7 + {}_{62}P_8$, or 138 848 807 594 160 ways to create a password with 6, 7, or 8 characters.

c) Use the indirect method. Then, there are $52^6 + 52^7 + 52^8$, or 54 507 570 843 648 ways to create a password with 6, 7, or 8 characters without a digit.
So, there are 221 918 520 426 688 – 54 507 570 843 648, or 167 410 949 583 040 ways to create a password with 6, 7, or 8 characters with at least one digit.

Chapter 2 Section 4 Question 16 Page 87

Suppose all seven numbers were distinct primes.
Multiplying any two numbers can be done in 7(6), or 42 ways. However, the number as factors can be arranged in 2! ways. So, there are 21 numbers created by multiplying two of the given numbers.
Multiplying any three numbers can be done in 7(6)(5), or 210 ways but only one sixth are unique. The number as factors can be arranged in 3! ways. So, there are 35 new numbers created by multiplying three of the given numbers.
Multiplying any four numbers can be done in 7(6)(5)(4), or 840 ways but the factors can be arranged in 4! ways. So, there are 35 new numbers created by multiplying four of the given numbers.
Similarly, there are $\frac{7(6)(5)(4)(3)}{5!}$, or 21 new numbers created by multiplying five of the given numbers.
Next, there are $\frac{7(6)(5)(4)(3)(2)}{6!}$, or 7 new numbers created by multiplying six of the given numbers.
Finally, there are $\frac{7!}{7!}$, or 1 new number created by multiplying all of the given numbers.
There would be a total of 120 different numbers formed by multiplying some or all of the numbers.

In this case, the seven numbers are a mix of prime and composite. So, repeated products must be eliminated. For example, $2 \times 6 = 3 \times 4 = 12$, $5 \times 6 = 5 \times 2 \times 3 = 30$, $5 \times 6 \times 8 = 5 \times 4 \times 3 \times 2 = 240$ should only be counted once. From the possible 120 products, 38 must be eliminated. There are a total of 82 different numbers formed by multiplying some or all of the numbers 2, 3, 4, 5, 6, 7 and 8.

Two-Factor Numbers		Three-Factor Numbers		Four-Factor Numbers		Five-Factor Numbers		Six-Factor Numbers	Seven-Factor Number
6	24	36	126	180	576	1152	2688	5760	40320
8	28	60	140	252	630	1260	2880	8064	
10	30	64	144	288	372	1440	3360	10 080	
12	32	70	160	320	720	1920	4032	13 440	
14	35	72	168	360	840	2016	5040	20 160	
15	40	80	192	384	960	2240	6720		
16	42	84	210	420	1008	2420			
18	48	90	224	448	1120				
20	56	96	240	480	1344				
21		105	280	504	1680				
		112	336	560					
		120							

Chapter 2 Section 4 Question 17 Page 87

a) The derangements of {1, 2, 3, 4} are {2, 1, 4, 3}, {2, 3, 4, 1}, {2, 4, 1, 3}, {3, 1, 4, 2}, {3, 4, 1, 2}, {3, 4, 2, 1}, {4, 1, 2, 3}, {4, 3, 1, 2}, and {4, 3, 2, 1}. There are 9 derangements.

b) Internet research shows that there are 44 derangements of {1, 2, 3, 4, 5}. The number of derangements of n items is given by $!n = (n - 1)[!(n - 1) + !(n - 2)]$.
For $n = 5$,
$!n = (n - 1)[!(n - 1) + !(n - 2)]$
$!5 = (5 - 1)[!(5 - 1) + !(5 - 2)]$
$!5 = 4[!4 + !3]$
$!5 = 4[9 + 2]$
$!5 = 44$

Chapter 2 Section 4 Question 18 Page 87

a) This is a derangement problem. Use the formula from the solution to question 17, part b).
For $n = 6$,
$!n = (n - 1)[!(n - 1) + !(n - 2)]$
$!6 = (6 - 1)[!(6 - 1) + !(6 - 2)]$
$!6 = 5[!5 + !4]$
$!6 = 5[44 + 9]$
$!6 = 265$
There are 265 ways that none of the cans will be labelled correctly.

b) Use the indirect method. First, calculate the total number of ways the labels can be replaced: 6!.
So, there are 6! – 265, or 455 ways at least one of the cans will be labelled correctly.

c) There is only one way for all the cans to be labelled correctly.

Chapter 2 Section 5 Probability Problems Using Permutations

Chapter 2 Section 5 Example 1 Your Turn Page 89

a) These trials are independent, so the probability of each person writing the same number is $\frac{1}{100}$. There are five trials, so

$$\begin{aligned} P(\text{all same}) &= \left(\frac{1}{100}\right)^5 \\ &= \frac{1}{10\,000\,000\,000} \end{aligned}$$

The probability that all numbers are the same is $\frac{1}{10\,000\,000\,000}$.

b) These trials are independent, so the probability of rolling a six on a die is $\frac{1}{6}$. There are five trials, so

$$\begin{aligned} P(\text{all 6s}) &= \left(\frac{1}{6}\right)^5 \\ &= \frac{1}{7776} \end{aligned}$$

For independent trials, $P(\text{all the same}) = (P(\text{a success}))^{\#\text{ trials}}$.

Chapter 2 Section 5 Example 2 Your Turn Page 90

The trials are dependent, since a person cannot be selected more than once. Use factorials.
$n(S) = 4!$
There is only one successful outcome, the single order of their grades, so $n(A) = 1$.

$$\begin{aligned} P(\text{in grade order}) &= \frac{1}{4!} \\ &= \frac{1}{24} \end{aligned}$$

The probability that the students will be order of their grades is $\frac{1}{24}$.

Chapter 2 Section 5 Example 3 Your Turn Page 91

a) Since Kylie selects the cards without replacement, the trials are dependent: $n(S) = {}_{52}P_5$.
Select three aces followed by two jacks: $n(A) = {}_4P_3 \times {}_4P_2$.

$$P(\text{ace, ace, ace, jack, jack}) = \frac{{}_4P_3 \times {}_4P_2}{{}_{52}P_5}$$
$$= \frac{24 \times 12}{311\,875\,200}$$
$$= \frac{288}{311\,875\,200}$$
$$= \frac{1}{1\,082\,900}$$

The probability of selecting three aces followed by two jacks is $\frac{1}{1\,082\,900}$.

b) Since Kylie selects the cards without replacement, the trials are dependent: $n(S) = {}_{52}P_5$.
Select two hearts followed by three clubs: $n(A) = {}_{13}P_2 \times {}_{13}P_3$.

$$P(\text{heart, heart, club,club, club}) = \frac{{}_{13}P_2 \times {}_{13}P_3}{{}_{52}P_5}$$
$$= \frac{156 \times 1716}{311\,875\,200}$$
$$= \frac{267\,696}{311\,875\,200}$$
$$= \frac{143}{166\,600}$$

The probability of selecting two hearts followed by three clubs is $\frac{143}{166\,600}$.

Chapter 2 Section 5 Example 4 Your Turn Page 92

a) There are 365 days in one year. So, for the sample space, each of the 16 people has 365 choices for their birthday. There are 16 trials, so $n(S) = 365^{16}$.
If everyone must have a different birthday, the successful outcomes are dependent and 16 days are selected from 365 days. So, $n(A) = {}_{365}P_{16}$.

$$P(\text{all different}) = \frac{{}_{365}P_{16}}{365^{16}}$$
$$\approx 0.7164$$

The probability that no two people have the same birthday is approximately 0.7164.

b) Use the indirect method, $P(A) = 1 - P(A')$.

$$P(\text{at least two the same}) = 1 - P(\text{no two the same})$$
$$= 1 - \frac{{}_{365}P_{16}}{365^{16}}$$
$$\approx 0.2836$$

The probability that at least two people have the same birthday is approximately 0.2836.

Chapter 2 Section 5 R1 Page 93

Using the method shown in Example 4,

$$\begin{aligned} P(\text{at least two the same}) &= 1 - P(\text{no two the same}) \\ &= 1 - \frac{{}_{365}P_{27}}{365^{27}} \\ &\approx 0.6269 \end{aligned}$$

The probability that at least two people have the same birthday is approximately 0.6269.
I would not accept the bet.

Chapter 2 Section 5 R2 Page 93

Answers may vary.
If the trials are dependent, permutations can be used. Look for restrictions such as, "without replacement" or "alphabetical order."

Chapter 2 Section 5 R3 Page 93

Answers may vary.

$$\begin{aligned} {}_{12}P_3 &= \frac{12!}{9!} \\ &= 12 \times 11 \times 10 \end{aligned} \qquad \begin{aligned} {}_{12}P_1 \times 3 &= \frac{12!}{11!} \times 3 \\ &= 12 \times 3 \end{aligned}$$

The first represents 3 of 12 objects being arranged. The second is 3 times 1 of 12 objects being arranged.

Chapter 2 Section 5 Question 1 Page 93

Since the student selects the cards without replacement, the trials are dependent: $n(S) = {}_{52}P_3$.
Select king, queen, jack: $n(A) = {}_4P_1 \times {}_4P_1 \times {}_4P_1$.

$$\begin{aligned} P(\text{king, queen, jack}) &= \frac{{}_4P_1 \times {}_4P \times {}_4P_1}{{}_{52}P_3} \\ &= \frac{4 \times 4 \times 4}{132\,600} \\ &= \frac{64}{132\,600} \\ &= \frac{8}{16\,575} \end{aligned}$$

The probability of selecting a king, a queen, and a jack is $\frac{8}{16\,575}$.

Chapter 2 Section 5 Question 2 Page 93

The trials are dependent, since a person cannot run the race more than once. Use factorials.
$n(S) = 6!$
The number of successful outcomes, Abby and Chantral will be the first two finishers, is $n(A) = 2! \times 4!$.

$$P = \frac{2! \times 4!}{6!}$$
$$= \frac{48}{720}$$
$$= \frac{1}{15}$$

The probability that Abby and Chantral will be the first two finishers is $\frac{1}{15}$.

Chapter 2 Section 5 Question 3 Page 93

The trials are dependent, since a name cannot be selected more than once: $n(S) = {}_{25}P_5$.
There is only one successful outcome, the names in alphabetical order, so $n(A) = 1$.

$$P(\text{in alphabetic order}) = \frac{1}{{}_{25}P_5}$$

Answer A.

Chapter 2 Section 5 Question 4 Page 93

There are 6 results for each roll. So, the sample space for the four trials is $n(S) = 6^4$.
The number of successful outcomes, a number divisible by three, is $n(A) = 2^4$.

$$P(\text{divisble by 3}) = \frac{2^4}{6^4}$$
$$= \left(\frac{1}{3}\right)^4$$
$$= \frac{1}{81}$$

Answer C.

Chapter 2 Section 5 Question 5 Page 93

The trials are dependent, since a ball cannot fall in more than once. Use factorials: $n(S) = 15!$
The number of successful outcomes, falling in order from 1 to 15, is $n(A) = 1$.

$$P(A) = \frac{1}{15!}$$
$$= \frac{1}{1\,307\,674\,368\,000}$$
$$P(A') = 1 - \frac{1}{15!}$$
$$= \frac{1\,307\,674\,367\,999}{1\,307\,674\,368\,000}$$

So, the odds against the balls following in order from 1 to 15 are

$$\frac{1\,307\,674\,367\,999}{1\,307\,674\,368\,000} : \frac{1}{1\,307\,674\,368\,000}.$$

Chapter 2 Section 5 Question 6 Page 93

a) and **b)** There are $_{365}P_3$ ways of choosing 3 days in a year, and $_{30}P_3$ ways of choosing 3 days from the 30 days of April.

$$\frac{_{30}P_3}{_{365}P_3} \approx 0.000\,505$$

Chapter 2 Section 5 Question 7 Page 94

a) Recall the table of all possible outcomes for the sum of the dice when two standard dice are thrown. There are a total of 36 outcomes with 6 having the successful outcomes of doubles.

$$P(A) = \frac{n(A)}{n(S)}$$

$$\begin{aligned} P(\text{doubles}) &= \frac{6}{36} \\ &= \frac{1}{6} \end{aligned}$$

The probability of rolling doubles with two dice is $\frac{1}{6}$.

b) These are independent events, so multiply the probabilities.

$$\begin{aligned} P(\text{doubles twice}) &= \frac{1}{6} \times \frac{1}{6} \\ &= \frac{1}{36} \end{aligned}$$

The probability of rolling doubles twice with two dice is $\frac{1}{36}$.

c) Recall the table of all possible outcomes for the sum of the dice when two standard dice are thrown. There are also 6 ways to roll a sum of 7. So, the probability of rolling consecutive sums of 7 on two rolls of dice is the same as the probability of rolling consecutive doubles.

Chapter 2 Section 5 Question 8 Page 94

a) These are independent events, so multiply the probabilities.

$$\begin{aligned} P(3\text{ boys}) &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

The probability that a family of three children has all boys is $\frac{1}{8}$.

b) These are independent events, so multiply the probabilities.

$$P(4\text{ boys}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{16}$$

The probability that a family of four children has all boys is $\frac{1}{16}$.

c) These are independent events, so multiply the probabilities.

$$P(5\text{ boys}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{32}$$

The probability that a family of five children has all boys is $\frac{1}{32}$.

d) These are independent events, so multiply the probabilities.

$$P(n\text{ boys}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \cdots \times \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{2^n}$$

The probability that a family of *n* children has all boys is $\frac{1}{2^n}$.

Chapter 2 Section 5 Question 9 Page 94

a) These are dependent events. The probability of selecting M first is $\frac{1}{9}$. The probability of selecting A second is $\frac{1}{8}$. The probability of selecting T third is $\frac{1}{7}$. The probability of selecting H fourth is $\frac{1}{6}$.

$$P(\text{MATH}) = \frac{1}{9} \times \frac{1}{8} \times \frac{1}{7} \times \frac{1}{6}$$
$$= \frac{1}{3024}$$

The probability that it spells MATH is $\frac{1}{3024}$.

b) These are dependent events. The probability of selecting one of the letters M, A, T, or H is $\frac{4}{9}$. The probability of selecting another letter from the set is $\frac{3}{8}$, and another letter is $\frac{2}{7}$, and the remaining letter is $\frac{1}{6}$.

$$P(\text{M,A,T,H}) = \frac{4}{9}\times\frac{3}{8}\times\frac{2}{7}\times\frac{1}{6}$$
$$= \frac{24}{3024}$$
$$= \frac{1}{126}$$

The probability that it includes the letters M, A, T, and H is $\frac{1}{126}$.

c) Since the letters are selected without replacement, the trials are dependent: $n(S) = {}_9P_4$.
Calculate the number of successful outcomes, contains an M. Use the indirect method.
The total number of results without restrictions is ${}_9P_4$, or 3024. The number of results that do not contain M is: 8(7)(6)(5), or 1680.
So, there are 3024 – 1680, or $n(A) = 1344$ ways to include the letter M.

$$P = \frac{1344}{3024}$$
$$= \frac{4}{9}$$

The probability that it includes the letter M is $\frac{4}{9}$.

Chapter 2 Section 5 Question 10 Page 94

a) The cards have to be in ascending order but they don't have to be consecutive.
The are 13 consecutive denominations in a deck: 2 3 4 5 6 7 8 9 10 J Q K A.
There are 4 choices for each denomination.
There are 7 possible sequences of 7 consecutive cards:
2 3 4 5 6 7 8
3 4 5 6 7 8 9
4 5 6 7 8 9 10
…
8 9 10 J Q K A

So, $\frac{{}_{13}C_7 \times 4^7}{{}_{52}P_7} \approx 4.1697\times10^{-5}$.
The probability of that the cards are dealt in ascending order is approximately 4.1697×10^{-5}.

b) Since the players are each dealt a card without replacement, the trials are dependent:
$n(S) = {}_{52}P_7$.
"Denomination," means one of the thirteen symbols that is represented on each card: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A. Note there are four of each in a standard deck.
Select cards that are not of the same denomination: $n(A) = 52(48)(44)(40)(36)(32)(28)$.

$$P(\text{no same denomination}) = \frac{52(48)(44)(40)(36)(32)(28)}{{}_{52}P_7}$$
$$\approx 0.2102$$

The probability of dealing no cards of the same denomination is approximately 0.2102.

Chapter 2 Section 5 Question 11 Page 94

There are 365 days in one year. So, for the sample space, each of the 20 people has 365 choices for their birthday. There are 20 trials, so $n(S) = 365^{20}$.
If everyone must have a different birthday, the successful outcomes are dependent and 20 days are selected from 365 days. So, $n(A) = {}_{365}P_{20}$.

$$\begin{aligned} P(\text{all different}) &= \frac{{}_{365}P_{20}}{365^{20}} \\ &\doteq 0.5886 \end{aligned}$$

Use the indirect method, $P(A) = 1 - P(A')$.

$$\begin{aligned} P(\text{at least two the same}) &= 1 - P(\text{no two the same}) \\ &= 1 - \frac{{}_{365}P_{20}}{365^{20}} \\ &\approx 0.4114 \end{aligned}$$

The probability that at least two people have the same birthday is approximately 0.4114.

Chapter 2 Section 5 Question 12 Page 94

From Example 4, the probability that at least two students have the same birthday in 30 students is approximately 0.7063. From question 11, the probability that at least two students have the same birthday in 20 students is approximately 0.4414. So, the number of students will be between 20 and 30. Use trial and error.

Try $n = 25$. Then, $P(\text{at least two the same}) \doteq 0.5687$. Too high.

Try $n = 22$. Then, $P(\text{at least two the same}) \doteq 0.4757$. Too low.

Try $n = 23$. Then, $P(\text{at least two the same}) \doteq 0.5073$.

So, 23 students are needed for the probability that at least two people have the same birthday reaches 0.5.

Chapter 2 Section 5 Question 13 Page 94

a) Since the songs are played without repeats, the events are dependent.

The probability of playing your most favourite song first is $\frac{1}{10}$, your second most favourite next is $\frac{1}{9}$, and so on.

$$\begin{aligned} P(\text{songs in order}) &= \frac{1}{10} \times \frac{1}{9} \times \frac{1}{8} \times \cdots \times \frac{1}{2} \times 1 \\ &= \frac{1}{3\,628\,800} \end{aligned}$$

The probability that the songs are played in your order of preference is $\frac{1}{3\,628\,800}$.

b) The probability of playing one of your two favourite songs first is $\frac{2}{10}$, and the probability of playing the other of your two favourite songs next is $\frac{1}{9}$.

$$P = \frac{2}{10} \times \frac{1}{9}$$
$$= \frac{2}{90}$$
$$= \frac{1}{45}$$

The probability that your two favourite songs are first and second is $\frac{1}{45}$.

Chapter 2 Section 5 Question 14 Page 94

a) The trials are independent, since the card is replaced: $n(S) = 52^5$.
Use the complement. The number of unsuccessful outcomes, no two people choose the same card, is $n(A') = 52(51)(50)(49)(48)$.

$$P(A') = \frac{52(51)(50)(49)(48)}{52^5}$$
$$\approx 0.8203$$

Then, $P(A) \approx 0.1797$.

So, the odds against at least two people choosing the same card are 0.8203:0.1797.

b) By "denomination," means one of the thirteen symbols that is represented on each card: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A. Note there are four of each in a standard deck.

The trials are independent, since the card is replaced: $n(S) = 52^5$.
Use the complement. The number of unsuccessful outcomes, no two people choose the same denomination, is $n(A) = 52(48)(44)(40)(36)$.

$$P(A') = \frac{52(48)(44)(40)(36)}{52^5}$$
$$\approx 0.4160$$

Then, $P(A) \approx 0.5840$.

So, the odds against at least two people choosing the same denomination are 0.4160:0.5840.

Chapter 2 Section 5 Question 15 Page 94

The table shows the results of 10 trials in Microsoft® Excel.

Trial	Mode
1	230
2	none
3	17
4	292
5	308
6	175
7	none
8	243
9	338
10	none

	A	B
1	Random Number	
2	272	
3	348	
4	64	
5	25	
6	223	
7	48	
8	20	
9	106	
10	228	
11	215	
12	202	
13	232	
14	342	
15	316	
16	226	
17	165	
18	77	
19	256	
20	230	
21	246	
22	306	
23	54	
24	282	
25	177	
26	364	
27	27	
28	291	
29	143	
30	66	
31	230	
32	Mode	230

The number of classes in which at least two people share the Same birthday is 7.

Chapter 2 Section 5 Question 16 Page 94

a) i) Since the numbers must be different, the trials are dependent: $n(S) = {}_{35}P_5$.
The number of successful outcomes, correct combination, is $n(A) = 1$.

$$\begin{aligned} P(A) &= \frac{1}{{}_{35}P_5} \\ &= \frac{1}{38\,955\,840} \end{aligned}$$

The probability of cracking a combination lock on a safe if five different numbers are used from 1 to 35 is $\frac{1}{38\,955\,840}$.

ii) Since the numbers must be different, the trials are dependent: $n(S) = {}_{40}P_5$.
The number of successful outcomes, correct combination, is $n(A) = 1$.

$$\begin{aligned} P(A) &= \frac{1}{{}_{40}P_5} \\ &= \frac{1}{78\,960\,960} \end{aligned}$$

The probability of cracking a combination lock on a safe if five different numbers are used from 1 to 40 is $\frac{1}{78\,960\,960}$.

iii) Since the numbers must be different, the trials are dependent: $n(S) = {}_{45}P_5$.
The number of successful outcomes, correct combination, is $n(A) = 1$.

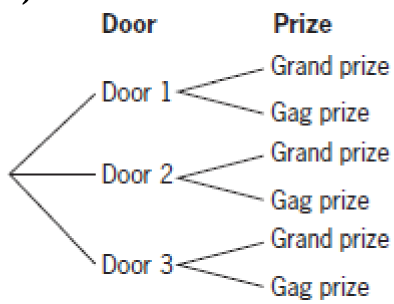
$$P(A) = \frac{1}{_{45}P_5}$$
$$= \frac{1}{146\,611\,080}$$

The probability of cracking a combination lock on a safe if five different numbers are used from 1 to 45 is $\frac{1}{146\,611\,080}$.

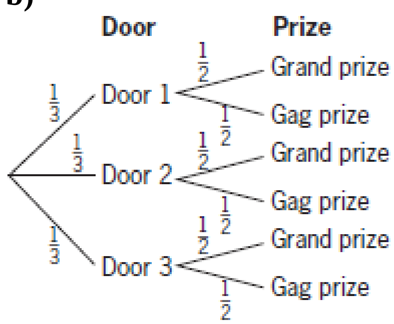
b) The probability of cracking the safe decreases as the five different numbers are chosen from a greater range of number.

Chapter 2 Section 5 Question 17 Page 95

a)



b)



c) Using the tree diagram, the probability of winning the grand prize is $\frac{1}{3} \times \frac{1}{2}$, or $\frac{1}{6}$.

d) Using the tree diagram, the probability of winning a good prize is $\frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2}$, or $\frac{1}{3}$.

e) Using the tree diagram, the probability of winning a gag prize is $\frac{1}{3}\times\frac{1}{2}$ + $\frac{1}{3}\times\frac{1}{2}$ + $\frac{1}{3}\times\frac{1}{2}$, or $\frac{1}{2}$.

f) The sum of all the probabilities is $\frac{1}{6}+\frac{1}{3}+\frac{1}{2}$, or 1. There are only three possibilities for prizes, so the sum of all probabilities in the sample space must equal 1.

Chapter 2 Section 5 Question 18 Page 95

Choose the date as your birthday. Then, the problem is the same.
There are 365 days in one year. So, for the sample space, each of the 25 people has 365 choices for their birthday. There are 25 trials, so $n(S) = 365^{25}$.
If everyone must have a different birthday, the successful outcomes are dependent and 25 days are selected from 365 days. So, $n(A) = {}_{365}P_{25}$.

$$\begin{aligned} P(\text{all different}) &= \frac{{}_{365}P_{25}}{365^{25}} \\ &\approx 0.4313 \end{aligned}$$

Use the indirect method, $P(A) = 1 - P(A')$.

$$\begin{aligned} P(\text{at least two the same}) &= 1 - P(\text{no two the same}) \\ &= 1 - \frac{{}_{365}P_{25}}{365^{25}} \\ &\approx 0.5687 \end{aligned}$$

The probability that at least two people have the same birthday is approximately 0.5687.

Chapter 2 Section 5 Question 19 Page 95

a) Recall the table of all possible outcomes for the sum of the dice when two standard dice are thrown. There are only 6 ways to roll a sum of 7 and 30 ways to not roll a sum of 7. So, it is more likely that not throwing a sum of 7 on consecutive rolls will occur.

b) The probability of arranging five digits in ascending order is $\frac{1}{5}\times\frac{1}{4}\times\frac{1}{3}\times\frac{1}{2}\times1$. The probability of three letters being arranged in alphabetical order is $\frac{1}{3}\times\frac{1}{2}\times1$. So, it is more likely that three different letters being arranged in alphabetical order will occur.

c) The probability of at least two out of 20 friends having the same birthday is approximately 0.4414. See solution to question 11.

There are 12 months in one year. So, for the sample space, each of the five people has 12 choices for their birth month. There are five trials, so $n(S) = 12^5$.
If everyone must have a different birth month, the successful outcomes are dependent and five months are selected from 12 months. So, $n(A) = {}_{12}P_5$.

$$\begin{aligned} P(\text{all different}) &= \frac{{}_{12}P_5}{12^5} \\ &\approx 0.3819 \end{aligned}$$

The probability that at least two out of five friends have the same birth month is approximately 1 – 0.3819, or 0.6181.
So, it is more likely that two out of five friends having the same birth month will occur.

Chapter 2 Section 5 Question 20 Page 95

Since the numbers must be different, the trials are dependent: $n(S) = {}_{40}P_5$.
The number of successful outcomes, all five numbers in the correct order, is $n(A) = 1$.

$$\begin{aligned} P(A) &= \frac{1}{{}_{40}P_5} \\ &= \frac{1}{78\,960\,960} \\ &\approx 1.2664 \times 10^{-8} \end{aligned}$$

The probability of winning the first prize is approximately 1.2664×10^{-8}.

The sample space is the same: $n(S) = {}_{40}P_5$.
The number of successful outcomes, four out of five numbers in correct order, is $n(A) = 4!$.

$$\begin{aligned} P(A) &= \frac{4!}{{}_{40}P_5} \\ &= \frac{24}{78\,960\,960} \\ &\approx 3.0395 \times 10^{-7} \end{aligned}$$

The probability of winning the second prize is approximately 3.0395×10^{-7}.
So, the probability of winning the first or second prize is approximately 3.1664×10^{-7}.

Chapter 2 Section 5 Question 21 Page 95

Answers may vary. Any scenario that has $n(A) = 1$ and $n(S) = {}_{15}P_7$. For example, winning first prize, similar to question 20.

Chapter 2 Section 5 Question 22 Page 95

a) There are a total of 170 points on the grid. The sample space is $n(S) = {}_{170}P_2$.
The number of successful outcomes, the segment is horizontal, is $n(A) = 17(16)(10)$. There are 17 points per row and 10 rows.

$$\begin{aligned} P(A) &= \frac{17(16)(10)}{{}_{170}P_2} \\ &\approx 0.0947 \end{aligned}$$

b) There are two diagonals, running from (0, 0) to (16, 9) and from (16, 0) to (0, 9).
The total number of line segments of length one or more is 170 × 169.

$$\begin{aligned}\text{P(line segment is on a diagonal)} &= \frac{\text{total number of diagonals}}{\text{total number line segments}}\\ &= \frac{2}{170\times 169}\\ &= \frac{2}{28730}\\ &\approx 6.9613\times 10^{-5}\end{aligned}$$

The probability that a segment is on a diagonal is about 6.9613×10^{-5}.

Chapter 2 Section 5 Question 23 Page 95

a) There are eight ways to win with cards in a row, a column, or a diagonal. If a card is picked, then the probability of the remaining two cards are of the same denomination is $\frac{3}{51}\times\frac{2}{50}$. So, the probability that there is exactly one winning set of the same denomination is $8\times\frac{3}{51}\times\frac{2}{50}$, or about 0.0188.

b) There are eight ways to win with cards in a row, a column, or a diagonal. If a card is picked, then the probability of the remaining two cards are consecutive in any order is $2\times\frac{4}{51}\times\frac{4}{50}$. So, the probability that there is exactly one winning set of consecutive cards is $8\times 2\times\frac{4}{51}\times\frac{4}{50}$, or about 0.1004.

Chapter 2 Section 5 Question 24 Page 95

a) The are 13 consecutive denominations in a deck: 2 3 4 5 6 7 8 9 10 J Q K A.
There are 4 choices for each denomination.
There are 8 possible sequences of 6 consecutive cards:
2 3 4 5 6 7
3 4 5 6 7 8
4 5 6 7 8 9
…
9 10 J Q K A

$$\begin{aligned}P(\text{consecutive and in order}) &= 8\times\frac{4^6}{{}_{52}P_6}\\ &\approx 2.2355\times 10^{-6}\end{aligned}$$

b) If the cards can be in any order, then there are 6! permutations of the six cards.

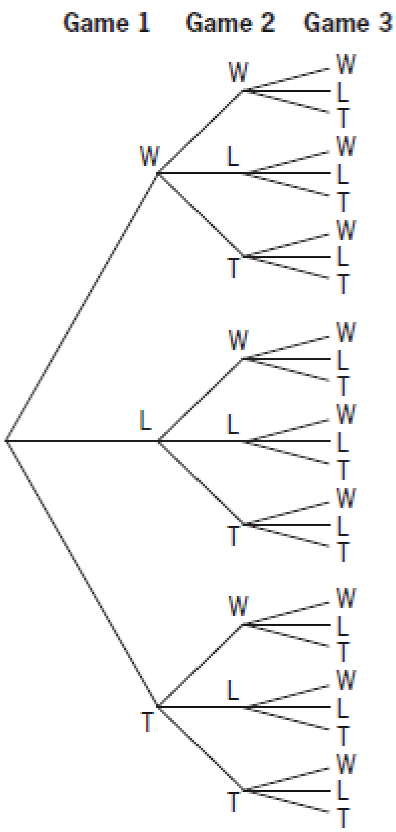
$$\begin{aligned}P(\text{consecutive in any order}) &= 8\times\frac{6!\times 4^6}{{}_{52}P_6}\\ &\approx 0.0026\end{aligned}$$

The probability that the six cards are consecutive, but in any order is $6! \times 3.6327 \times 10^{-6}$, or about 0.0026.

Chapter 2 Review

Chapter 2 Review **Question 1** **Page 96**

From one team's perspective. There are 27 possible outcomes.



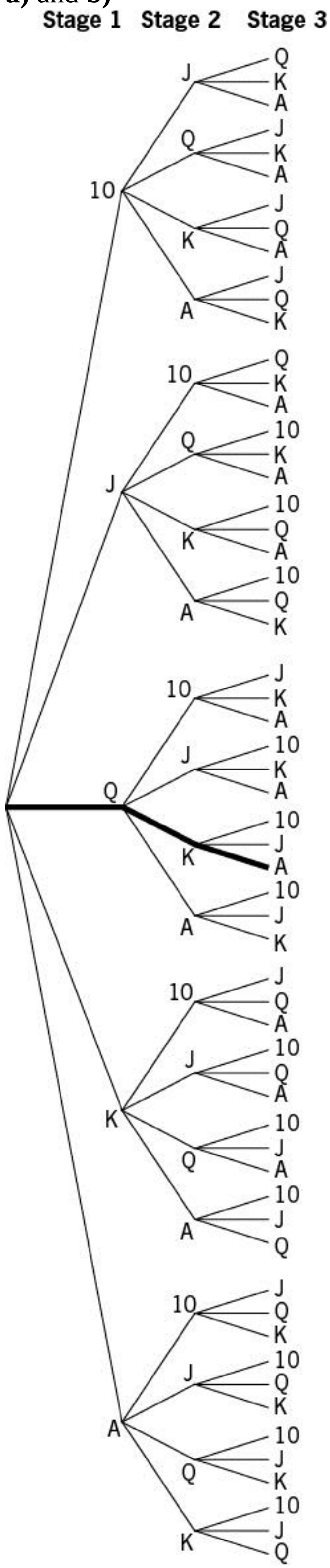
Chapter 2 Review **Question 2** **Page 96**

Second Die \ First Die	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12
5	6	7	8	9	10	11	12	13
6	7	8	9	10	11	12	13	14
7	8	9	10	11	12	13	14	15
8	9	10	11	12	13	14	15	16

The sum of 9 occurs eight times. There is only one occurrence of the sum 2 and sum 16.

Chapter 2 Review Question 3 Page 96

a) and **b)**



c) There are 60 possible outcomes.

Chapter 2 Review Question 4 Page 96

a) Assuming repetition is permitted, there are 10^5, or 100 000 unique five-digit security codes possible.

b) At 8 s per try, it will take Sarah at most 800 000 s, or about 9.3 days to find the correct code.

Chapter 2 Review Question 5 Page 96

a) Ryan has 3(4)(5)(3)(2), or 360 choices to configure his computer.

b) If there were six choices for the video card, then Ryan has 3(4)(6)(3)(2), or 432 choices to configure his computer. Increasing the number of choices for any option will increase the total number of possible configurations.

Chapter 2 Review Question 6 Page 97

Assume that the socks have three stripes. Barb can make 6(5)(5), or 150 distinct pairs of socks.

Chapter 2 Review Question 7 Page 97

A company can assign three jobs to five employees in ${}_5P_3$, or 60 ways.

Chapter 2 Review Question 8 Page 97

a)

```
            1
         2     2
      3     6     6
   4    12    24    24
5    20    60   120   120
```

b)

```
               1
            2     2
         3     6     6
      4    12    24    24
   5    20    60   120   120
6    30   120   360   720   720
```

The first term in row *n* is *n*. To obtain the remaining terms in row *n*, multiply all the terms in the row above by *n*.

c) Answers may vary. The last term in row *n* equals *n*!. The last two terms in each row are equal.

Chapter 2 Review Question 9 Page 97

Consider each group a book. These can be arranged 3! ways. Then, the novels can be arranged in 7! ways, the plays in 4! ways, and the poetry books in 5! ways. So, the books can be arranged in $3! \times 7! \times 4! \times 5!$, or 87 091 200 ways.

Chapter 2 Review Question 10 Page 97

a) There are 3 vowels and 4 consonants in the word STORAGE. If the vowels must remain in even positions, there are 4(3)(3)(2)(2)(1)(1), or 144 ways this can be done.

b) Since there are fewer vowels than odd positions, there are several possible cases for vowel positions:1, 3, 5; 1, 3, 7; 3, 5, 7; and 1, 5, 7. If the vowels must remain in odd positions, there are $4 \times 3(4)(2)(3)(1)(2)(1)$, or 576 ways this can be done.

c) If the vowels can be in even or odd positions, there are 7!, or 5040 ways this can be done.

Chapter 2 Review Question 11 Page 97

Use the complement. There are 6! ways to arrange the six letters. The number of unsuccessful outcomes, the vowels are together, is $4! \times 3!$. Then, the number of ways that the vowels will not be together is $6! - (4! \times 3!)$, or 576 ways.

Chapter 2 Review Question 12 Page 97

Since the ten thousands place cannot contain 0, there are several cases to consider for a five-digit even number formed by using all the digits 0, 1, 2, 3, and 4.
Case 1: 0 in the ones place
There is one choice for the ones, four choices for the tens place, three choices for the hundreds place, two choices for the thousands, and one choice for the ten thousands: 1(2)(3)(4)(1), or 24 ways.

Case 2: 0 in the tens place
There are two choices for the ones place, one choice for the tens place, three choices for the hundreds place, two choices for the thousands, and one choice for the ten thousands: 1(2)(3)(1)(2), or 12 ways.

Case 3: 0 in the hundreds place
There are two choices for the ones place, three choices for the tens place, one choice for the hundreds place, two choices for the thousands, and one choice for the ten thousands: 1(2)(1)(3)(2), or 12 ways.

Case 4: 0 in the thousands place
There are two choices for the ones place, three choices for the tens place, two choices for the hundreds place, one choice for the thousands, and one choice for the ten thousands: 1(1)(2)(3)(2), or 12 ways.

There are 24 + 12 + 12 + 12, or 60 ways to form a five-digit even number using all the digits 0, 1, 2, 3, and 4.

Chapter 2 Review Question 13 Page 97

a) The probability of each person selecting their own name is $\frac{1}{10}\times\frac{1}{9}\times\frac{1}{8}\times\cdots\times\frac{1}{3}\times\frac{1}{2}\times 1$, or approximately 2.7557×10^{-7}.

b) Use the complement. The probability that nobody selects his or her own name is $1 - 2.7557 \times 10^{-7}$, or about 1.

Chapter 2 Review Question 14 Page 97

a) The probability that Kendra selects the green ball and Abdul selects the red ball is $\frac{1}{6}\times\frac{1}{5}$, or $\frac{1}{30}$.

b) The probability that Kendra selects the green ball and Abdul does not select the red ball is $\frac{1}{6}\times\frac{4}{5}$, or $\frac{2}{15}$.

c) Use the complement. The probability that Kendra does not select the green ball and Abdul does not select the red ball is $1 - \frac{1}{30}$, or $\frac{29}{30}$.

Chapter 2 Review Question 15 Page 97

a) The probability that each person selects the same letter is $\frac{1}{26}\times\frac{1}{26}\times\frac{1}{26}\times\frac{1}{26}\times\frac{1}{26}$, or approximately 8.4165×10^{-8}.

b) Use the complement. The probability of each person selects a different letter is $1 - 8.4165 \times 10^{-8}$, or about 1.

Chapter 2 Test Yourself

Chapter 2 Test Yourself Question 1 Page 98

If a standard die is rolled four times, there are 6^4 possible orders of faces. Answer C.

Chapter 2 Test Yourself Question 2 Page 98

$$\begin{aligned} {}_{101}P_{98} &= \frac{101!}{(101-98)!} \\ &= \frac{101!}{3!} \end{aligned}$$

Answer D.

Chapter 2 Test Yourself   Question 3   Page 98

When flipping a coin five times, the probability that heads turns up every time is $\left(\frac{1}{2}\right)^5$, or $\frac{1}{32}$.

Answer A.

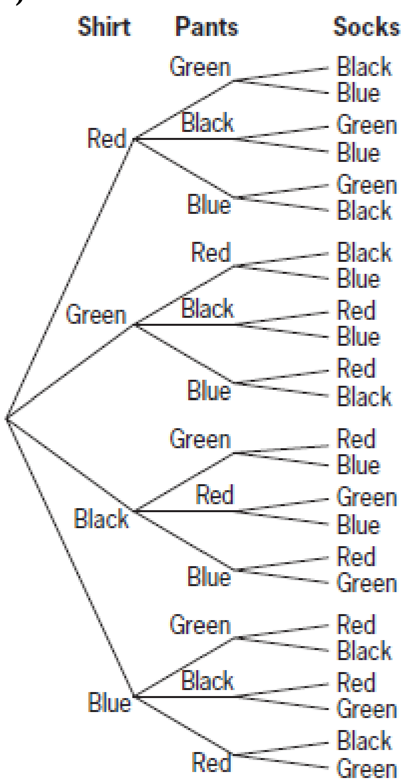
Chapter 2 Test Yourself   Question 4   Page 98

The expression ${}_9P_{10}$ is not defined, $n < r$.

$$
\begin{aligned}
{}_9P_{10} &= \frac{9!}{(9-10)!} \\
&= \frac{9!}{(-1)!}
\end{aligned}
$$

Chapter 2 Test Yourself   Question 5   Page 98

a)



b) Starting with a red pair of pants, Rosa has 6 choices.

Chapter 2 Test Yourself   Question 6   Page 98

The starting lineup can be chosen in 4(3)(4)(3)(4)(2), or 1152 ways.

Chapter 2 Test Yourself Question 7 Page 98

There are ${}_{12}P_5$, or 95 040 ways to assign five different roles in a play to the 12 members of a drama club.

Chapter 2 Test Yourself Question 8 Page 98

Since each Canadian can win either first, second, or third, the events are dependent.

There are three ways the Canadians can win gold, so the probability of winning gold is $\frac{3}{8}$.

There are two ways the Canadians can win silver, so the probability of winning silver is $\frac{2}{7}$.

There is one way the Canadians can win bronze, so the probability of winning bronze is $\frac{1}{6}$.

$$\begin{aligned}&\frac{3}{8}\times\frac{2}{7}\times\frac{1}{6}\\&=\frac{6}{336}\\&=\frac{1}{56}\end{aligned}$$

The probability that three Canadians will win gold, silver, and bronze is $\frac{1}{56}$, or approximately 0.0179.

Chapter 2 Test Yourself Question 9 Page 99

a) There are eight letters in the word COMPUTER. These can be arranged in 8!, or 40 320 ways.

b) There are five consonants in the word COMPUTER. The number of arrangements that begin with a consonant is (5)(7)(6)(5)(4)(3)(2)(1), or 25 200.

Chapter 2 Test Yourself Question 10 Page 99

First, determine the number ways the captain and assistant captain can be together.
Treat the captain and assistant captain as a single person, making 10 players. Then, arrange the two of them among themselves: 10! × 2!

Now, use the indirect method.

$$\begin{aligned}\text{Total} - \text{together} &= 11! - 10! \times 2!\\&= 39\,916\,800 - 7\,257\,600\\&= 32\,659\,200\end{aligned}$$

There are 32 659 200 ways the soccer team can line up if the captain and assistant captain must remain apart.

Chapter 2 Test Yourself Question 11 Page 99

a) There are 25 men and 20 women who belong to a club, for a total of 45 members. With no restrictions, the executive panel of four people can be chosen in ${}_{45}P_4$, or 3 575 880 ways.

b) Use the indirect method. Determine how many executives can be formed without women: $_{25}P_4$. Determine how many executives can be formed without men: $_{20}P_4$.

$$\begin{aligned}\text{Total} - \text{no women} - \text{no men} &= {}_{45}P_4 - {}_{25}P_4 - {}_{20}P_4 \\ &= 3\,575\,880 - 303\,600 - 116\,280 \\ &= 3\,156\,000\end{aligned}$$

There are 3 156 000 ways the executive panel can be formed with at least one women and one man.

c) For the executive panel to have a president and vice president of different genders, there are two cases: male president and female vice president or female president and male vice president. For the first case, the executive panel of four people can be chosen in (25)(20)(43)(42), or 903 000 ways. Similarly for the second case, the executive panel of four people can be chosen in (20)(25)(43)(42), or 903 000 ways.

There are 903 000 + 903 000 , or 1 806 000 ways the executive panel can be formed with a president and vice president of different genders.

Chapter 2 Test Yourself Question 12 Page 99

a) The probability of selecting A, B, C, D if repetition is permitted is $\left(\frac{1}{26}\right)^4$, or $\frac{1}{456\,976}$.

b) The probability of selecting A, B, C, D if repetition is not permitted is $\frac{1}{26}\left(\frac{1}{25}\right)\left(\frac{1}{24}\right)\left(\frac{1}{23}\right)$, or $\frac{1}{358\,800}$.

Chapter 2 Test Yourself Question 13 Page 99

First, determine the probability that no two select the same number.

The probability that each person selects a different number is $\frac{20}{20}\times\frac{19}{20}\times\frac{18}{20}\times\cdots\times\frac{12}{20}\times\frac{11}{20}$, or approximately 0.0655.

Then, use the complement. The probability that at least two people select the same number is 1 – 0.0655, or about 0.9345.

Chapter 2 Test Yourself Question 14 Page 99

a) When dealing a card to five players, there are $_{52}P_5$, or 311 875 200 different results possible.

b) When players must receive different denominations, there are 52(48)(44)(40)(36), or 158 146 560 results possible.

c) The probability that four players receive cards of the same denomination is $\frac{4}{52}\times\frac{3}{51}\times\frac{2}{50}\times\frac{1}{49}$, or approximately 3.6938×10^{-6}.

d) If players chose a card each from a full deck, the probability that four players receive cards of the same denomination is $\frac{4}{52}\times\frac{4}{52}\times\frac{4}{52}\times\frac{4}{52}$, or approximately 3.5013×10^{-5}.