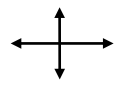
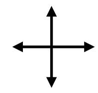
Unit 03: Practice Test

Part A: Answer in the space provided. One Mark Each.

1. Sketch the curves. a) y = $-e^x$  b) y = ln (-x) 

2. Solve the following.

a) $2\ln(e^{\frac{x}{2}}) = 5$ b) $e^{\frac{1}{3}\ln 8} = x$ c) q = ln 1 d) $\frac{dy}{dx} = 3y$ (give a function)

3. Evaluate the limits. a) $\lim_{x\to 0}(1+6x)^{\frac{1}{x}}$ b) $\lim_{x\to 0^+} e^{\ln(x)}$ c) $\lim_{x\to 0}\frac{\cos x - 1}{x}$ d) $\lim_{x\to 0}\frac{\sin x}{5x}$

4. Differentiate. a) $y = \ln x^5$ b) $y = e^{\sin x}$ c) $y = 12^x$

d) $f(x) = \cos 3x$ e) $f(x) = \sin^2 x$ f) $f(x) = \cot x$

5. Given $f(x) = e^{2x}$, determine the value of $f^{(11)}(1)$.

1. Differentiate the following. Do Not Simplify.

e) $y = (\sin x)(\ln x)$ b) $y = \left(e^x + \sqrt[3]{\cos x}\right)^7$ c) $y = \frac{\sqrt{1-\tan x}}{3x^3}$ d) $y = \ln\left(\tan^2 e^{2x}\right)$

2. Find the equation of the tangent to $y = \sin x \tan\frac{x}{2}$, when $x = \frac{\pi}{3}$ [5]

3. Show that if $f(x) = \ln\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)$, then $f'(x) = -\csc x$

APPS

1. The position of a certain oscillating (vibrating) object is given by $s = 8\cos(2t + \frac{\pi}{3})$.

   a) Determine the velocity and acceleration of the body. [2]
   b) What is the maximum velocity the particle will obtain? [1]
   c) What is the earliest time this maximum be obtained (t > 0) [1]
   d) What is the objects' position when it has maximum velocity? [1]

2. After *t* seconds, the electric charge *A* of a circuit decays according to the formula $A = A_0 e^{-\frac{t}{d}}$, where A_0 is the initial charge and *d* is a real constant. The initial charge of 10 units was reduced to $\frac{10}{e}$ units in 3 sec. Find the rate of change of charge at 3 seconds.

Communication. Level:________

The table of values is for the functions, $Y_1 = \ln(x)$ and $Y_2 = \ln(ex)$.

a) Fill in the blanks in the first two columns. **Show your work.**
b) Fill in the column for Y_2 without a calculator. Explain why this was an easy task.
c) Calculate Y_1' and Y_2'. Show that they are equal. Explain why this makes sense.

X	Y1	
1	0	
2	.69315	
3	1.0986	
4	1.3863	
10	2.3026	
17	2.8332	

X=

TIPS Level___________

1. The population *P* of a certain species of animal is given by $P = e^{-at}$, where *a* is constant and *t* is time in years. Show that the rate of change of population of this species is $-aP$.

2. Prove that $y = \sec x + \tan x$ is always increasing on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

3. You have a picture of the function y = r^x and its derivative. Which is which? What is the value of r?