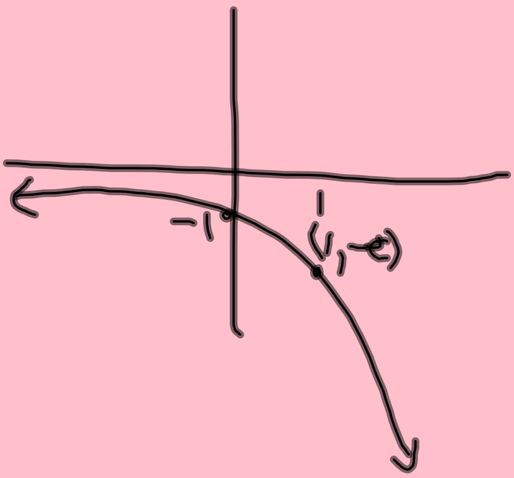
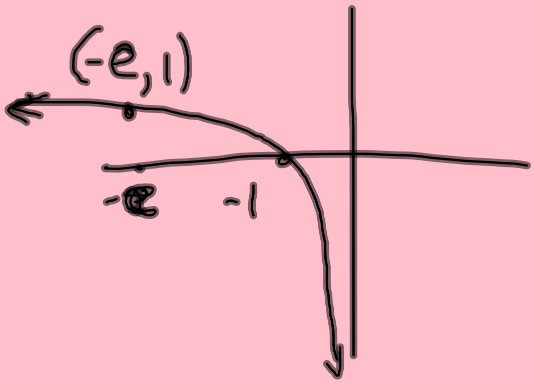
1. Sketch the curves. a) $y = -e^x$ b) $y = \ln(-x)$





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2. Solve the following.

a) $2\ln(e^{\frac{x}{2}}) = 5$

$\ln e^{x/2} = 5/2$

$e^{x/2} = e^{5/2}$

$e^x = e^5$

$x = 5$

b) $e^{\frac{1}{3}\ln 8} = x$

$1/3 \ln 8 = \ln x$

$\cancel{\ln} 8^{1/3} = \cancel{\ln} x$

$8^{1/3} = x$

$2 = x$

2. Solve the following.

c) q = ln 1

$e^q = 1$

$q = 0$

d) $\frac{dy}{dx} = 3y$ (give a function)

$y = e^{3x}$

LS	RS
$\frac{dy}{dx}$	$3y$
$= e^{3x} \cdot 3$	$= 3(e^{3x})$

LS = RS

3. Evaluate the limits

a) $\lim_{x\to 0}(1+6x)^{\frac{1}{x}}$ b) $\lim_{x\to 0^+} e^{\ln(x)}$

$* \lim_{x\to 0}(1+x)^{\frac{1}{x}}$
$= e$

$\lim_{x\to 0}\left[(1+6x)^{\frac{1}{6x}}\right]^6$

$= e^6$

$= \lim_{x\to 0^+} x$

$= 0$

3. Evaluate the limits.

c) $\lim_{x\to 0}\frac{\cos x - 1}{x}$

$= 0$

$$\boxed{\lim_{x\to 0}\frac{A^h - 1}{h} = \ln A}$$

d) $\lim_{x\to 0}\frac{\sin x}{5x}$

$= \lim_{x\to 0}\left(\frac{\sin x}{x}\right)\left(\frac{1}{5}\right)$

$= (1)\left(\frac{1}{5}\right)$

$= \frac{1}{5}$

4. Differentiate. a) $y = \ln x^5$ b) $y = e^{\sin x}$ c) $y = 12^x$

a) $y' = \left(\frac{1}{x^5}\right)\left(5x^4\right)$ b) $y' = (\cos x)(e^{\sin x})$

c) $y' = (\ln 12)(12^x)$

4. Differentiate

d) $f(x) = \cos 3x$ e) $f(x) = \sin^2 x$ f) $f(x) = \cot x$

$f'(x) = -\sin(3x)3$ $f'(x) = 2\sin x(\cos x)$

f) $f(x) = -\csc x \cot x$

5. Given $f(x) = e^{2x}$, determine the value of $f^{(11)}(1)$.

$f'(x) = 2(e^{2x})$

$f''(x) = 2^2(e^{2x})$

$f'''(x) = 2^3(e^{2x})$

$f^{(4)}(x) = 2^4(e^{2x})$

$f^{(11)}(x) = 2^{11}(e^{2x})$

$f^{(11)}(1) = 2^{11}(e^{2(1)})$

$f^{(11)}(1) = 2048(e^2)$

1. Differentiate the following. Do Not Simplify.

$$y = (\sin x)(\ln x)$$

$$y' = \sin x\left(\frac{1}{x}\right) + (\ln x)\cos x$$

1. Differentiate the following. Do Not Simplify.

$$y = \left(e^x + \sqrt[3]{\cos x}\right)^7$$

$$y' = 7\left(e^x + (\cos x)^{\frac{1}{3}}\right)^6 \cdot \left(e^x + \frac{1}{3}(\cos x)^{-\frac{2}{3}} \cdot -\sin x\right)$$

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1. Differentiate the following. Do Not Simplify.

$$y = \frac{\sqrt{1-\tan x}}{3x^3}$$

$$y' = \frac{\frac{1}{2}(1-\tan x)^{-\frac{1}{2}} \cdot (-\sec x^2)(3x^2) - 9x^2 \cdot (1-\tan x)^{\frac{1}{2}}}{9x^6}$$

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1. Differentiate the following. Do Not Simplify.

$$y = \ln\left(\tan^2 e^{2x}\right)$$

$$y' = \frac{1}{\tan^2 e^{2x}} \cdot 2\tan(e^{2x}) \cdot \sec^2(e^{2x}) \cdot 2e^{2x}$$

Find the equation of the tangent to $y = \sin x \tan \frac{x}{2}$, when $x = \frac{\pi}{3}$ [5]

$y' = \cos x \tan\frac{x}{2} + \frac{1}{2}\sec^2\frac{x}{2}\sin x$

$y'(\frac{\pi}{3}) = \sqrt{3}/2$

$y = \sqrt{3}/2\, x + b$

$(1/2) = \sqrt{3}/2(\pi/3) + b$

$(1/2 - \frac{\sqrt{3}\pi}{6}) = b$

Find $P_T(\frac{\pi}{3}, y)$

$\hookrightarrow y = 1/2$

$P_T(\frac{\pi}{3}, \frac{1}{2})$

$\therefore y_T = \frac{\sqrt{3}}{2}x + \left(\frac{3 - \sqrt{3}\pi}{6}\right)$

Show that if $f(x)=\ln\left(\sqrt{\dfrac{1+\cos x}{1-\cos x}}\right)$, then $f'(x)=-\csc x$

$$f(x)=\ln\left(\frac{1+\cos x}{1-\cos x}\right)^{\frac{1}{2}}$$

$$=\frac{1}{2}\ln(1+\cos x)-\frac{1}{2}\ln(1-\cos x)$$

$$f'(x)=\frac{1}{2}\frac{1}{1+\cos x}\cdot(\sin x)-\frac{1}{2}\frac{1}{1-\cos x}\cdot\sin x$$

$$=\frac{1}{2}\left[\frac{-\sin x}{1+\cos x}-\frac{\sin x}{1-\cos x}\right]$$

$$=\frac{1}{2}\left[\frac{-\sin x+\cancel{\sin x\cos x}-\sin x-\cancel{\sin x\cos x}}{(1-\cos^2 x)}\right]$$

$$=\frac{-\cancel{2}\cancel{\sin x}}{\cancel{2}(\sin^2 x}$$

APPS. The position of a certain oscillating (vibrating) object is given by

$$s = 8\cos(2t + \frac{\pi}{3})$$

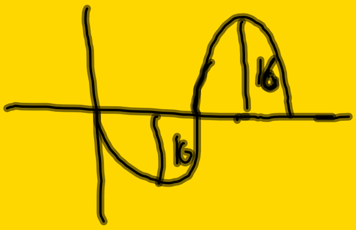
Determine the velocity and acceleration of the body.

What is the maximum velocity the particle will obtain?

$V(t) = 8(-\sin(2t + \frac{\pi}{3}))\cdot 2$

$a(t) = 16(-\cos(2t + \frac{\pi}{3}))\cdot 2$

$-16\sin(2t + \frac{\pi}{3})$

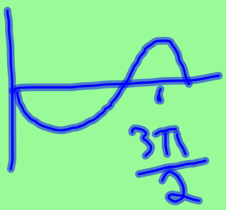


APPS. The position of a certain oscillating (vibrating) object is given by

$$s = 8\cos(2t + \frac{\pi}{3})$$

What is the earliest time this maximum be obtained (t > 0)
What is the objects' position when it has maximum velocity?

c) $V = -16\sin(2t + \frac{\pi}{3})$



$2t + \frac{\pi}{3} = \frac{3\pi}{2}$

$2t = \frac{9\pi}{6} - \frac{2\pi}{6}$

$2t = \frac{7\pi}{6}$

$t = \frac{7\pi}{12}$

d) $S\left(\frac{7\pi}{12}\right) =$

. After t seconds, the electric charge A of a circuit decays according to the formula $A = A_0 e^{-\frac{t}{d}}$, where A_0 is the initial charge and d is a real constant. The initial charge of 10 units was reduced to $\frac{10}{e}$ units in 3 sec. Find the rate of change of charge at 3 seconds.

$$A = A_0 e^{-\frac{1}{d}t}$$

$$A_0 = 10$$

$$A(3) = \frac{10}{e}$$

$$A = 10e^{-\frac{1}{d}t}$$

$$\frac{\cancel{10}}{e} = \cancel{10}e^{-\frac{3}{d}}$$

$$\frac{1}{e} = e^{-\frac{3}{d}}$$

$$\ln\frac{1}{e} = -\frac{3}{d}$$

$$-1 = -\frac{3}{d}$$

$$\boxed{d = 3}$$

$$A = 10e^{-\frac{1}{3}t}$$

$$A' = 10e^{-\frac{1}{3}t}\cdot\left(-\frac{1}{3}\right)$$

$$A'(3) = 10e^{-1}\left(-\frac{1}{3}\right)$$

$$= \frac{-10}{3e}$$

$$= 10\left(\frac{1}{3}\right)\left(\frac{-\ln 3}{3}\right)$$

$$\boxed{= -\frac{10}{9}\ln 3}$$

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Communication

The table of values is for the functions, $Y_1 = \ln(x)$ and $Y_2 = \ln(ex)$.

a) Fill in the blanks in the first two columns. **Show your work.**

b) Fill in the column for Y_2 without a calculator. Explain why this was an easy task.

c) Calculate Y_1' and Y_2'. Show that they are equal. Explain why this makes sense.

X	Y1	Y2
1	0	1
2	.69315	1.69315
3	1.0986	2.0986
4	1.3863	2.3863
10	2.3026	3.3026
17	2.8332	
X=█		3.8332

$$\ln(ex) = \ln x + \ln e$$
$$= \ln x + 1$$

$$y_1 = \ln x \qquad y_2 = \ln(ex)$$
$$y_1' = \frac{1}{x} \qquad y_2' = \frac{1}{\cancel{e}x} \cdot \cancel{e}$$
$$= \frac{1}{x}$$

$$y = \ln(5x)$$
$$y' = \frac{1}{\cancel{5}x} \cdot \cancel{5}$$

TIPS Level__________

1. The population P of a certain species of animal is given by $P = e^{-at}$, where a is constant and t is time in years. Show that the rate of change of population of this species is $-aP$.

$$P = e^{-at}$$

$$P = e^{-at} \cdot -a$$

$$P = -aP$$

2. Prove that $y = \sec x + \tan x$ is always increasing on $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$.

$$y = \sec x + \tan x$$
$$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$$
$$= \frac{1 + \sin x}{\cos x}$$
$$y' = \frac{\cos^2 x - (1+\sin x)(-\sin x)}{\cos^2 x}$$
$$= \frac{\cos^2 x + \sin x + \sin^2 x}{\cos^2 x}$$
$$= \frac{1 + \sin x}{\cos^2 x}$$

Denominator is never negative

$1 + \sin x > 0$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$,

since the derivative of the original function

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3. You have a picture of the function $y = r^x$ and its derivative. Which is which?
What is the value of r?