Sample Test: Solutions

Unit 04 Test

K/U

1) Determine an expression for $\frac{dy}{dx}$. $3(y-2)^3 - x^3 = 4x - 6$. [2]

$3 \cdot 3(y-2)^2 \cdot \frac{dy}{dx} - 3x^2 = 4$

$9(y-2)^2 \cdot \frac{dy}{dx} = 4 + 3x^2$

$\frac{dy}{dx} = \frac{4+3x^2}{9(y-2)^2}$

2) What is the absolute min and absolute max of the following function on [-5,2].

$f(t) = t^3 - 12t + 2$ [3]

$f'(t) = 3t^2 - 12$

$0 = 3t^2 - 12$

$12 = 3t^2$

$4 = t^2$

$t = \pm 2$

$f(2) = -14$

$f(-2) = 18$ max

$f(-5) = -63$ min

3) Determine the absolute maximum and minimum values of $f(x) = x^2e^x$ on $x \in [-5, 1]$. [4]

$f'(x) = 2xe^x + x^2e^x$

$0 = 2xe^x + x^2e^x$

$0 = x \cdot e^x(2+x)$

$x = 0, -2$

$f(0) = 0$ min

$f(-2) = 0.54$

$f(-5) = 0.17$

$f(1) = 2.7$ max

$\uparrow$

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4) A particle is moving along the x-axis according to $s(t) = t + \sin(t)$ where t is in seconds, $t \geq 0$, and s(t) is in meters. [6]

a) Give the velocity function of the particle in terms of t.

$s'(t) = v(t)$

$= 1 + \cos t$

b) How fast is the particle traveling at π seconds?

$s'(\pi) = v(\pi) = 1 + \cos\pi = 1 + (-1) = 0$ m/s

c) What is the average velocity in the first 2π seconds?

$\frac{s(2\pi) - s(0)}{2\pi} = \frac{2\pi + \sin 2\pi - \sin 0}{2\pi} = \frac{2\pi}{2\pi} = 1$ m/s

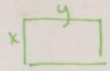
d) When is the particle at rest?

$0 = 1 + \cos t$

$-1 = \cos t$

$t = \pm\pi, 3\pi, 5\pi \ldots$

5) The perimeter of a rectangle is 36 m. Use a calculus approach to determine the maximum area of such a rectangle. [4]



$P = 2x + 2y$

$36 = 2x + 2y$

$36 - 2y = 2x$

$\frac{36-2y}{2} = x$

$18 - y = x$

$A = xy$

$= \cancel{\ }$

$= (18-y)y$

$A'(y) = (-1)y + (18-y)$

$= 18 - 2y$

$0 = 18 - 2y$

$9 = y$

$x = 18 - 9$

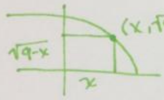
$= 9$

$A = 9 \times 9$

$= 81 m^2$

APPS

6) What is the area of the largest rectangle that has its base on the x-axis, its lower left corner at (0, 0) and its upper right corner on the graph of $f(x) = \sqrt{9-x}$? [5]



$A = x \cdot \sqrt{9-x}$

$A'(x) = \sqrt{9-x} + x \cdot \frac{1}{2}(9-x)^{-1/2} \cdot (-1)$

$= \sqrt{9-x} - \frac{x}{2\sqrt{9-x}}$

$0 = \sqrt{9-x} - \frac{x}{2\sqrt{9-x}}$

$0 = \frac{2(9-x) - x}{2\sqrt{9-x}}$

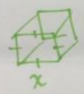
$0 = 2(9-x) - x$

$0 = 18 - 3x$

$x = 6$

$A(6) = 6\sqrt{3}$

7) An open top box, with a square base is being designed. Material costs 12¢/cm^2. What is the largest box that can be built for \$51.84. [5]



$SA = x^2 + 4xy$

$cost = 0.12(x^2 + 4xy)$

$51.84 = 0.12(x^2 + 4xy)$

$51.84 = 0.12x^2 + 0.48xy$

$\frac{51.84 - 0.12x^2}{0.48x} = y$

$A = x^2 y$

$= x^2\left(\frac{51.84 - 0.12x^2}{0.48x}\right)$

$= x(108 - 0.25x^2)$

$A' = (108 - 0.25x^2) + x(-0.5x)$

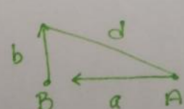
$= 108 - 0.75x^2$

$0 = 108 - 0.75x^2$

$x = \pm 12 \quad y = 6 \quad V = 12^2 \cdot 6$

$= 864 cm^3$

8) Car A is 40 km east of Car B and begins moving west at 40 km/h. At the same moment, Car B begins to move north at 70 km/h. What is the closest distance in kilometres the cars will be from each other and at what time t, in hours, will that distance occur? [5]



$d = \sqrt{(40-40t)^2 + (70t)^2}$

$= \sqrt{1600 - 3200t + 6500t^2}$

$d'(t) = \frac{1}{2}(1600 - 3200t + 6500t^2)^{-1/2} \cdot (-3200 + 13000t)$

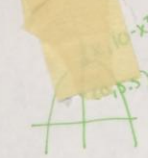
$0 = -3200 + 13000t$

$t \doteq 0.25h$

$d(0.25) = 34.7km$ at 15 min

TIPS

9) Find the point on the parabola $y = 10 - x^2$ closest to the point $(0, 5.5)$. [6]



$P_1(0,5.5)$

$P_2(x, 10-x^2)$

$d = \sqrt{(x-0)^2 + (10-x^2-5.5)^2}$

$= \sqrt{x^2 + (4.5-x^2)^2}$

$d'(x) = \frac{1}{2}(x^2+(4.5-x^2)^2)^{-1/2} \cdot (2x + 2(4.5+x^2)\cdot(-2x1))$

$0 = (2x + 2(4.5-x^2)(-2x))$

$= 4x^3 - 16x$

$= 4x(x^2-4)$

$x = 0, \pm 2$

$d(0) = 4.5$

$d(\pm 2) = 2.1$ * min $\therefore$ both $x = \pm 2$

10) A company is producing Netbook computers. In this manufacturing process, the number of defective computers that must be rejected tends to increase as the daily output increases. The number of rejects **r** depends on the total daily output, **x**, according to the equation: $r(x) = \dfrac{60x}{250-x}$, for $x \leq 180$ where 180 is the maximum possible output. Each computer produced is either sold or rejected. The company makes a profit of \$300 for each computer sold but loses \$100 for each one rejected. [6]

a) What is the profit if they produce the maximum number of computers?

$P = 300x - 100(r(x))$

$= 300x - 100\left(\frac{650x}{250-x}\right)$

$P(180) = 300(180) - 100\left(\frac{60(180)}{250-180}\right)$

$= \$38\ 571.43$

b) What output will maximize the profit?

$P'(x) = 300 - 100\left(\frac{60(250-x) - 60x(-1)}{(250-x)^2}\right)$

$0 = 300 - 100\left(\frac{310 - 60x + 60x}{(250-x)^2}\right)$

$300 = \frac{31000}{(250-x)^2}$

$(250-x)^2 = \frac{31000}{3000}$

$-x = \sqrt{\frac{31}{3}} - 250$

$x \doteq 239.83$

$\therefore$ output should be 240