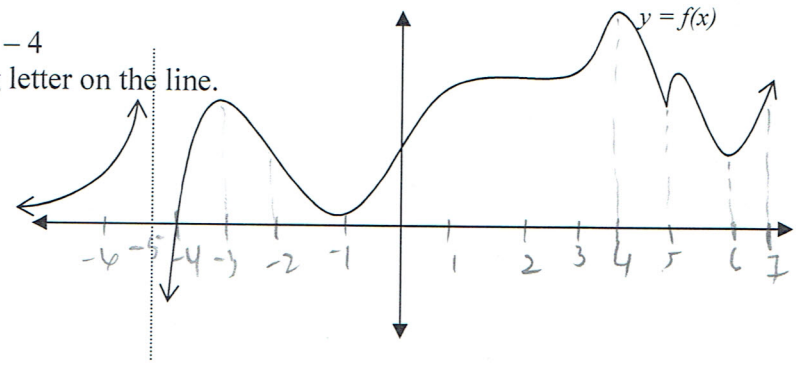
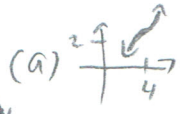
Part A: Knowledge and Understanding. Mark_____/

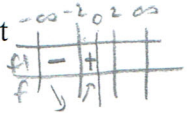
Use the graph y = $f(x)$ to answer questions 1 – 4



1. For each section place the corresponding letter on the line.
   A) = 0 B) < 0 C) > 0 D) undefined
   a) $f(6)$ >0
   b) $f(-4)$ =0
   c) $f'(2)$ <0 decreasing
   d) $f'(8)$ >0
   e) $f''(-5)$ DNE
   f) $f'(2)$ <0 (concave down)
2. State the value(s) of **a** (an integer) if:
   a) $f(a) < 0$, $f'(a) > 0$ and $f''(a) < 0$ −5, 5
   b) $f(a) > 0$, $f'(a) > 0$, and $f''(a) = 0$ 0
3. On the interval [-8, 8], in how many places is $f'(x)$ undefined? two x=-5, 5
4. On the interval [-8, 8], what is the largest interval of increase? (-1, 4)
5. a) Sketch g(x) in the neighbourhood of x = 4, if g(4) = 2, $g'(4) = 1$, & $g''(4) = 0$.
   b) Sketch h(x) around x = 1, if h(1) = 1, $h'(1) = -1$ and $h''(1) < 0$.



6. Is the stationary point x = -2, of the function $f(x) = x^4 - 8x^2$ a local max, a local min or a point of inflection? [2] $f'(x) = 4x^3 - 16x = 4x(x^2 - 4)$ max



7. If (-3, 6) is a point on an even function, give another point on the function. (3, 6)
8. What is the y-intercept of an odd function with no discontinuities? 0
9. Determine the intercepts, local maximums and minimums, and points of inflection of the curve y = $10x^3 - 3x^2$. Use the points to sketch the curve. [7]

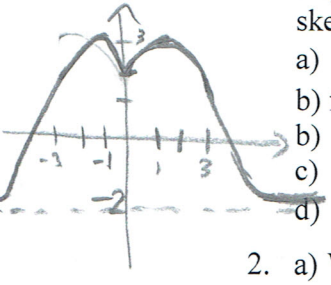
APPS

1. Analyze the curve $f(x) = \frac{5x}{1+x}$ under the seven headings. Then sketch the curve.

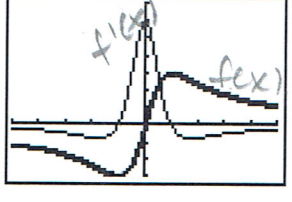
(Domain-Intercepts-Symmetry-Asymptotes-Inc/Dec-Max/Min-Concavity)

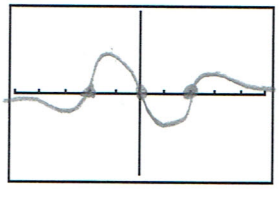
Communication Level_____

1. Explain what each of these symbolic statements says about the graph of f(x). Then sketch **one function** that satisfies each of the 5 conditions. The first is done for you.
   a) Domain = $\{x \mid x \in \Re\}$ and f(x) is continuous. The function has no breaks.
   b) f(-x) = f(x) even function
   b) f(1) = 3, $f'(1) = 0$ (1, 3) & x=1 is a critical point
   c) $f''(x) < 0$, $0 < x < 3$, and $f''(x) > 0$, $x > 3$ concave down (0, 3) & concave up (3, ∞) x=3 inflexion point
   d) $\lim_{x \to \infty} f(x) = -2$ HA at x = -2



2. a) Which is the function and which is the derivative? Explain.
   b) Sketch the second derivative f'' on the empty grid.





TIPS : Level :_____

1. The function $y = ax^4 + bx^2 + c$ has slope -16 at its point of inflection (1, -5).

Determine the values of a, b, and c.

PaAA

(9) $y = 10x^3 - 3x^2 = x^2(10x - 3)$

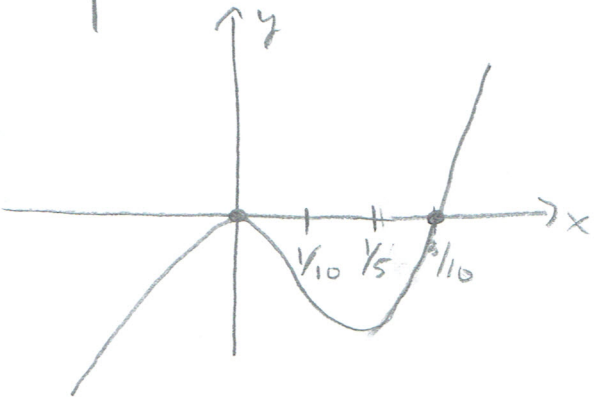
Intercepts: $(0,0)$ $(3/10, 0)$

$y' = 30x^2 - 6x = 6x(5x-1)$ $x = 0, x = 1/5$ critical points

$y'' = 60x - 6 = 6(10x-1)$ $x = 1/10$ inflexion point

	$-\infty$		0		$1/5$		∞
$f'(x)$		$+$		$-$		$+$	
$f(x)$		$\nearrow$		$\searrow$		$\nearrow$	

	$-\infty$		$1/10$		∞
$f''(x)$		$-$		$+$	
$f(x)$		$\cap$		$\cup$	



Apps

(1) $f(x) = \dfrac{5x}{1+x}$ $D_f = \mathbb{R} \setminus \{-1\}$

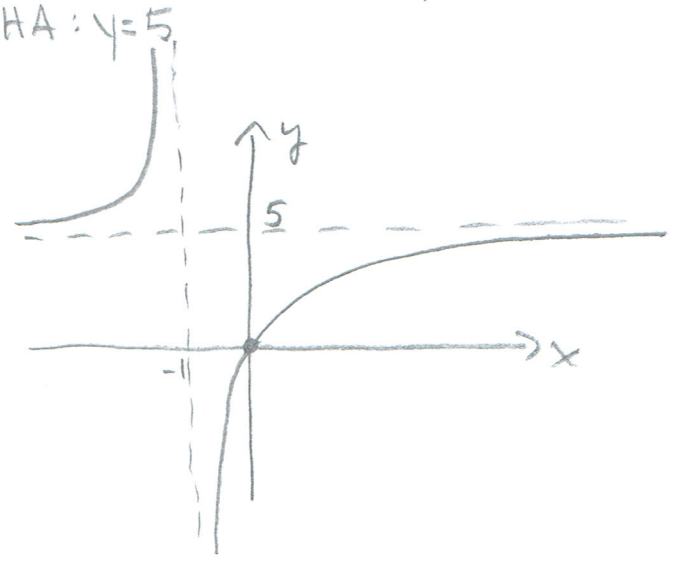
VA $x = -1$ HA: $y = 5$

Intercepts $(0,0)$

$f'(x) = \dfrac{5(1+x) - 5x}{(1+x)^2} = \dfrac{5}{(1+x)^2}$

$f''(x) = \dfrac{-10}{(1+x)^3}$

	$-\infty$		-1		∞
$f'(x)$		$+$		$+$	
$f''(x)$		$+$		$-$	
$f(x)$		$\nearrow \cup$		$\nearrow \cap$	



TIPS

① $y = ax^4 + bx^2 + c$

slope $= -16$

$f'(1) = -16$

Inflexion pt: $(1, -5)$

$f''(1) = 0$

$y' = 4ax^3 + 2bx$

$\rightarrow y'(1) = 4a + 2b = -16$

$y'' = 12ax^2 + 2b$

$\rightarrow y''(1) = 12a + 2b = 0$

$6a + b = 0$

$\boxed{b = -6a}$

plug in

$2a + b = -8$

$2a - 6a = -8$

$-4a = -8$

$\boxed{a = 2}$

$b = -6(2)$

$\boxed{b = -12}$

So $y = 2x^4 - 12x^2 + c$

we have $(1, -5)$

$-5 = 2 - 12 + c$

$c = 5$

$\therefore \boxed{y = 2x^4 - 12x^2 + 5}$

Hilroy