Unit 7: Sample Test Name:_____

1. Do the following equations describe lines **(L)** or planes **(P)**?
   a) $[x, y] = [1, 1] + s[2, 3]$ Line
   b) $[x, y, z] = t[1, 2, 3] + s[3, 4, -3]$ Plane
   c) $4x - 5y + z = 7$ Plane

2. List two points on each object.
   a) $\frac{x-3}{1} = \frac{y+7}{-2} = \frac{z-5}{4}$ (3,-7,5) (4,-9,9)
   b) $2x - 5y + z = 10$ (0,0,10) (5,0,0)

3. What are the three possibilities for the intersection of a line and a plane? [2]

4. Determine the parametric equation of the line that passes through (2, -1, 3) and is perpendicular to the plane $x - 6y + 4z = 12$. [2]

5. Determine the scalar equation of the plane with x –intercept = 2, y-intercept = –1, and z -intercept = 3 [4]

6. Determine the distance from the point (2, -1, 5) to the plane $3x - 3y + 5z = 8$. [2]

7. If the following lines intersect, determine the point of intersection. Otherwise classify the lines. (i.e.

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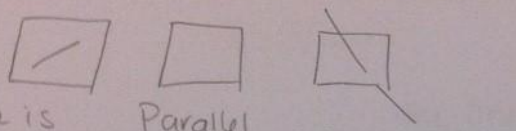
3)



Line is on plane | Parallel and distinct | one point of intersection

4) $P(2,-1,3)$ $\Pi: x-6y+4z=12$

$\vec{d} = (1,-6,4)$ $\vec{n} = (1,-6,4)$ ($\vec{n}$ is $\perp$ to Π) (so, $\vec{n}$ is a direction vector for Line)

$L: (2,-1,3)+t(1,-6,4)$

5) $A(2,0,0)$ $B(0,-1,0)$ $C(0,0,3)$

$\vec{AB} = (0,-1,0)-(2,0,0)$
$= (-2,-1,0)$

$\vec{AC} = (0,0,3)-(2,0,0)$
$= (-2,0,3)$

$\vec{d_1} = (-2,-1,0)$ $\vec{d_2} = (-2,0,3)$

$\vec{n} = (-2,-1,0) \times (-2,0,3)$
$= (-3, 6, -2)$

-2 -1 0
-2 0 3

$Ax+By+Cz+D=0$
$-3x+6y-2z+D=0$
$-3(2)+D=0$
$-6=-D$
$D=6$

$\boxed{-3x+6y-2z+6=0}$

6) $P(2,-1,5)$ $\Pi: 3x-3y+5z=8$

$x_1=2$ $y_1=-1$ $z_1=5$
$A=3$ $B=-3$ $C=5$ $D=-8$

$$d = \frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$$

$$= \frac{|3(2)+(-3)(-1)+5(5)+(-8)|}{\sqrt{3^2+3^2+5^2}}$$

$$= \frac{26}{\sqrt{43}}$$

7) $L_1: (x,y,z) = (0,-8,4) + t(3,1,-1)$ $\quad L_2: \frac{x-3}{1} = \frac{y+7}{-2} = \frac{z-5}{4}$

$\vec{d_1} = (3,1,-1)$ $\quad \vec{d_2} = (1,-2,4)$

the lines are not parallel

$L_1: x = 3t$
$y = -8+t$
$z = 4-t$

$L_2: x = 3+s$
$y = -7-2s$
$z = 5+4s$

at pt of int

① $3t = 3+s$ → ① $3t-3 = s$

② $-8+t = -7-2s$ → ② $-8+t = -7-2(3t-3)$

$t = -7-6t+6+8$

$t+6t = 7$

$7t = 7$

$t = 1$

① $3(1)-3 = s$

$0 = s$

Check in ③

$4-t = 5+4s$

LS: $4-1$ RS: $5+4(0)$

$3 \longleftrightarrow 5$

NOT SAME!

∴ the lines are skew.

8) $L_1: \frac{x-3}{-3} = \frac{y-8}{5}, z=4$ $\pi_1: 7x-2y+z-71=0$

$\vec{n}=(7,-2,1)$

$\vec{d}=(-3,5,0)$

$L_1: \begin{cases} x = 3-3t \\ y = 8+5t \\ z = 4. \end{cases}$

$7x-2y+z-71=0$

$7(3-3t)-2(8+5t)+4-71=0$

$21-21t-16-10t+4-71=0$

$-31t=62$

$t=-2$

∴ they intersect when $t=2$

$x=3-3(-2) = 9$

$y=8+5(-2) = -2.$

$z=4$

$(9,-2,4)$

Check

$7(9)-2(-2)+4-71$

$=0$ ☺

9) ① $x+2y+3z+4=0$ $\vec{n_1}=(1,2,3)$

② $x-y-3z-8=0$ $\vec{n_2}=(1,-1,-3)$

③ $x+5y+9z+16=0.$ $\vec{n_3}=(1,5,9)$

∴ none of the planes are ∥

Are they coplanar?

$\vec{n_1}\cdot(\vec{n_2}\times\vec{n_3})$

$=(1,2,3)\cdot((1,-1,-3)\times(1,5,9))$

$=(1,2,3)\cdot(6,-12,6)$

$=6-24+18$

$=0.$ Yes! coplanar

$(5+1, -3-9, -9+15)$

$=(6,-12,6)$

① $x+2y+3z+4=0$

$-$② $x-y-3z-8=0$

④ $3y+6z+12=0$

$+$⑤ $-3y-6z-12=0$

$0z=0$ ∴ they intersect in a line (star)

① $x+2y+3z+4=0$

$-$③ $x+5y+9z+16=0$

⑤ $-3y-6z-12=0$

9) cont. The system intersects in a line.

① $x + 2y + 3z + 4 = 0$

$-$ ② $x - y - 3z - 8 = 0$

$3y + 6z + 12 = 0$

$3y = -6z - 12$

$y = -2z - 6$

Set $z = t$

$\therefore$ they intersect in the line.

$x = 8 + t$
$y = -2t - 6$
$z = t$

$\vec{d}_1 = (1, -2, 1)$

① $x + 2y + 3z + 4 = 0$

$x + 2(-2t - 6) + 3(t) + 4 = 0$

$x - 4t - 12 + 3t + 4 = 0$

$x = 8 + t$

Check (just for fun!!)

① $x + 2y + 3z + 4 = 0$

$-$ ③ $x + 5y + 9z + 16 = 0$

$-3y - 6z = -12$

$-6z = -12 + 3y$

$z = 2 - \frac{1}{2}y$

Set $y = t$

$x = -10 - \frac{1}{2}t$
$y = t$
$z = 2 - \frac{1}{2}t$

$\vec{d}_2 = (-\frac{1}{2}, 1, -\frac{1}{2})$

$\vec{d}_1 = -2(\vec{d}_2)$
as required

① $x + 2y + 3z + 4 = 0$

$x + 2(t) + 3(2 - \frac{1}{2}t) + 4 = 0$

$x + 2t + 6 - \frac{3}{2}t + 4 = 0$

$x = -\frac{1}{2}t - 10$

10) A(0,0,0)

① $x+y+z=5$ $\quad \vec{n_1}=(1,1,1)$

\+ ② $3x+y-z=-7$ $\quad \vec{n_2}=(3,1,-1)$ not parallel int in a line

$4x+2y=-2$

$2y=-2-4x$

$y=-1-2x$

Set $x=t$

$x=t$
$y=-1-2t$
$z=6+t$

$\vec{d_1}=(1,-2,1)$

$B(0,-1,6)$

① $x+y+z=5$

$t+(-1-2t)+z=5$

$-t \quad z=6+t$

$\vec{AB}=(0,-1,6)-(0,0,0)$
$=(0,-1,6)$

$\Pi: (x,y,z)=(0,0,0)+s(0,-1,1)$
$+t(1,-2,6)$

11) a) ① $24=x+y+z$

$3.80=0.25x+0.10y+0.05z$ $\xrightarrow{\times 100}$ $380=25x+10y+5z$ ②

b) $10\times$① $240=10x+10y+10z$

\- ② $380=25x+10y+5z$

$-140=-15x+5z$

$\vec{n_1}=(1,1,1)$ not ||

$\vec{n_2}=(25,10,5)$ line inters

$x=t$
$y=52-4t$
$z=-28+3t$

$5z=-140+15x$

$z=-28+3x$

let $x=t$

$24=t+y+(-28)+3t$

$52-4t=y$

Actual solution

All three must be ≥ 0

$t\geq 0$, $52-4t\geq 0$, $-28+3t\geq 0$

$-4t\geq -52$ $\quad 3t\geq 28$

$t\leq 13$ $\quad t\geq \frac{28}{3}$

(div by -ve) $\quad \uparrow 9.3$

Possibilities:

$t=10,11,12,13$

$(10,12,2)$, $(11,8,5)$

$(12,4,8)$, $(13,0,11)$

FINAL ANSWER