Welcome to Calculus and Vectors!!!!!

https://www.youtube.com/watch?v=x5Y1BxtjkMc

WARM-UP

A very tall cylindrical pipe with a radius of 3 cm is filling with water at a constant rate of 27π cm^3/min. ($V = \pi r^2 h$)

a) Express the height of water in terms of time.
b) What is the average ROC of height of the water over the first 5 min?
c) How fast is the water rising at 3 minutes?

a) $V = \pi r^2 h$ $\quad 27\pi\ cm^3/min$

$= 27\pi \cdot t$

$$\frac{27\pi t}{9\pi} = \frac{\cancel{\pi(9)}h}{\cancel{9\pi}}$$

$$\boxed{3t = h}$$

b) $$AROC = \frac{h(5) - h(0)}{5 - 0} \quad cm/min$$

$$= \frac{15 - 0}{5}$$

$$= 3\ cm/min$$

c) $$IROC = \frac{h(3.01) - h(3)}{0.01}$$

$$= 3\ cm/min$$

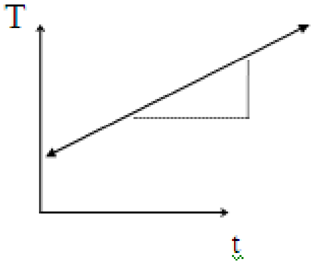
PART I: Rates of Change and the Derivative

Unit 1: Introduction to Calculus

Lesson 1: Average vs. Instant Rate of Change

Recall: Slope is a measure of rate of change.

Ex:



The temperature is rising at … °C/ min

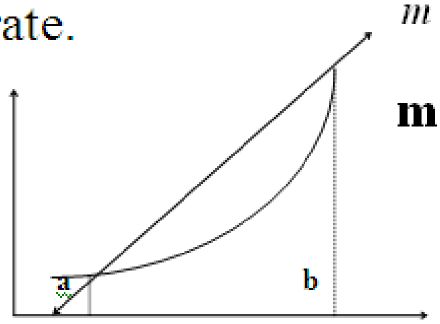
MUST include units

For linear relationships the rate of change is constant and equal to the slope of the line.

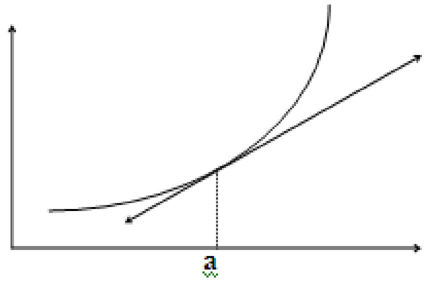
The rate of change of any relation can be approximated by the slope of the secant. This gives an **average** rate.

$$m = \frac{\Delta y}{\Delta x} \quad \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

m = average rate of change from **x** = a to **x** = b



The **instantaneous** rate of change at a point $x = \mathbf{a}$ is the slope of the tangent at that point.



m = instantaneous rate of change when **x** = a

Using function notation with $\mathbf{y} = f(\mathbf{x})$,

$$\text{Average rate} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$\text{Instantaneous rate} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$= \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

Example 1:

If $s(t) = 320-5t^2$

find IROC when t = 4

$$IROC = \lim_{h \to 0} \frac{s(4.01) - s(4)}{0.01}$$

$$= -40.05$$

Method 2:

$$IROC = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}$$

$$= \lim_{h \to 0} \frac{320-5(t+h)^2 - [320-5t^2]}{h}$$

$$= \lim_{h \to 0} \frac{320-5(t^2+2th+h^2)-320+5t^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{320}-\cancel{5t^2}-10th-5h^2\cancel{-320}\cancel{+5t^2}}{h}$$

$$= \lim_{h \to 0} \frac{-10(4)\cancel{h} - 5h^{\cancel{2}}}{\cancel{h}}$$

$$= \lim_{h \to 0} -40 - 5h$$

$$= -40$$

Example 2:

If $y = \sqrt{x+1} + 5$

find the equation of the tangent at x = 3

$$\lim_{h \to 0} \frac{\sqrt{x+h+1} + 5 - \left[\sqrt{x+1} + 5\right]}{h}$$

x = 3

$$= \lim_{h \to 0} \frac{\sqrt{4+h} + \cancel{5} - \sqrt{4} - \cancel{5}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h} \cdot \frac{\sqrt{4+h} + 2}{\sqrt{4+h} + 2}$$

$$= \lim_{h \to 0} \frac{\cancel{4} + h - \cancel{4}}{h\left[\sqrt{4+h} + 2\right]}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\ 1}{\cancel{h}\left[\sqrt{4+h} + 2\right]}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{4+h} + 2}$$

$$= \frac{1}{4}$$

diff. of squares

$(a-b)(a+b)$

$a^2 - b^2$