Example 2: Calculate slope of the secant to $y = 2x^3$ at the point where $x = 1$ and the second point is h units away.

$$m_1 = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{2(1+h)^3 - 2}{h}$$

$$= \lim_{h \to 0} \frac{2(1+2h+h^2)(1+h) - 2}{h}$$

$$= \lim_{h \to 0} \frac{2(1+h+2h+2h^2+h^2+h^3) - 2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2} + 6h^{\cancel{2}\,1} + 6\cancel{h} + 2h^{\cancel{3}\,2} - \cancel{2}}{\cancel{h}}$$

$$= \lim_{h \to 0} 6h + 6 + 2h^2$$

$$= 6$$

The slope of the tangent at x = a,

$$m_a = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Example 3: Calculate the slopes of the tangents to the curves at the given points.

a) $y = x^2$ at $x = 3$

b) $f(x)=\dfrac{2}{3x+2}$ at x = 1.

$$m_1 = \lim_{h\to 0} \frac{\frac{2}{3(1+h)+2} - \frac{2}{5}}{h}$$

$$= \lim_{h\to 0} \left[\frac{2}{5+3h} - \frac{2}{5}\right] \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \left[\frac{2(5) - 2(5+3h)}{(5+3h)(5)}\right] \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{10} - \cancel{10} - 6\cancel{h}}{25+15h} \cdot \frac{1}{\cancel{h}}$$

$$= \lim_{h\to 0} \frac{-6}{25+15h}$$

$$= \frac{-6}{25}$$

c) $y = x^3 - 4x$ at $x = 0$

$$m_0 = \lim_{h \to 0} \frac{h^3 - 4h - (\cancel{x^3} - \cancel{4x})}{h}$$

$$= \lim_{h \to 0} \frac{h^{\cancel{3}\,2} - 4\cancel{h}}{\cancel{h}}$$

$$= \lim_{h \to 0} h^2 - 4$$

$$= -4$$

Example 2: Sketch the curve $y = x^3 - 4x$, and its tangent at $x = 0$.

$y = x^3 - 4x = x(x^2-4)$

$\boxed{}$ $= x(x-2)(x+2)$

degree 3

L.C = + D/U