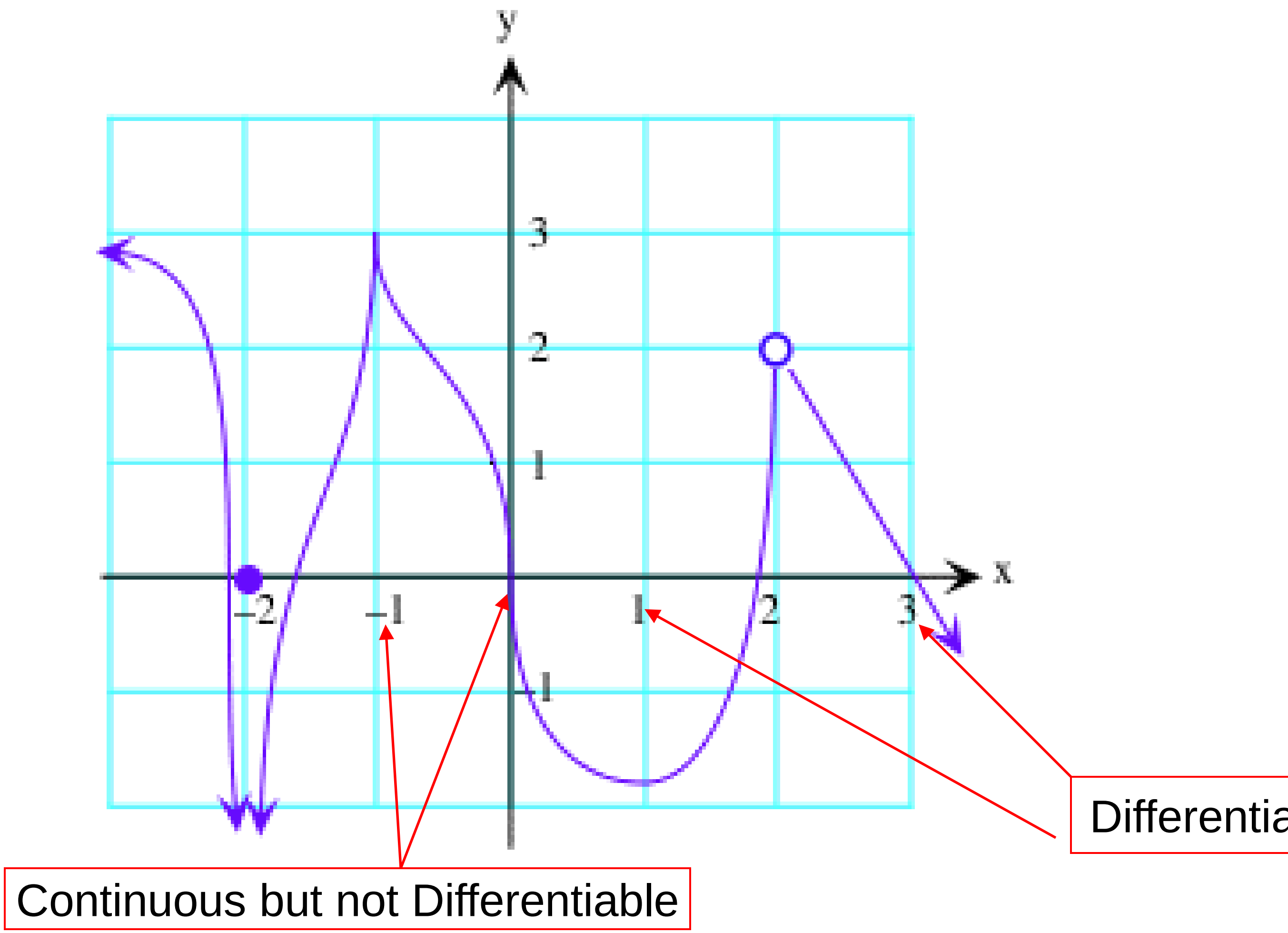
Graphical Representation



Differentiable

- f is differentiable at point "a" if f'(a) exists

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

- f is differentiable on the subset of its domain if it is differentiable at each point of the subset.

Undifferentiable

- f is not differentiable when

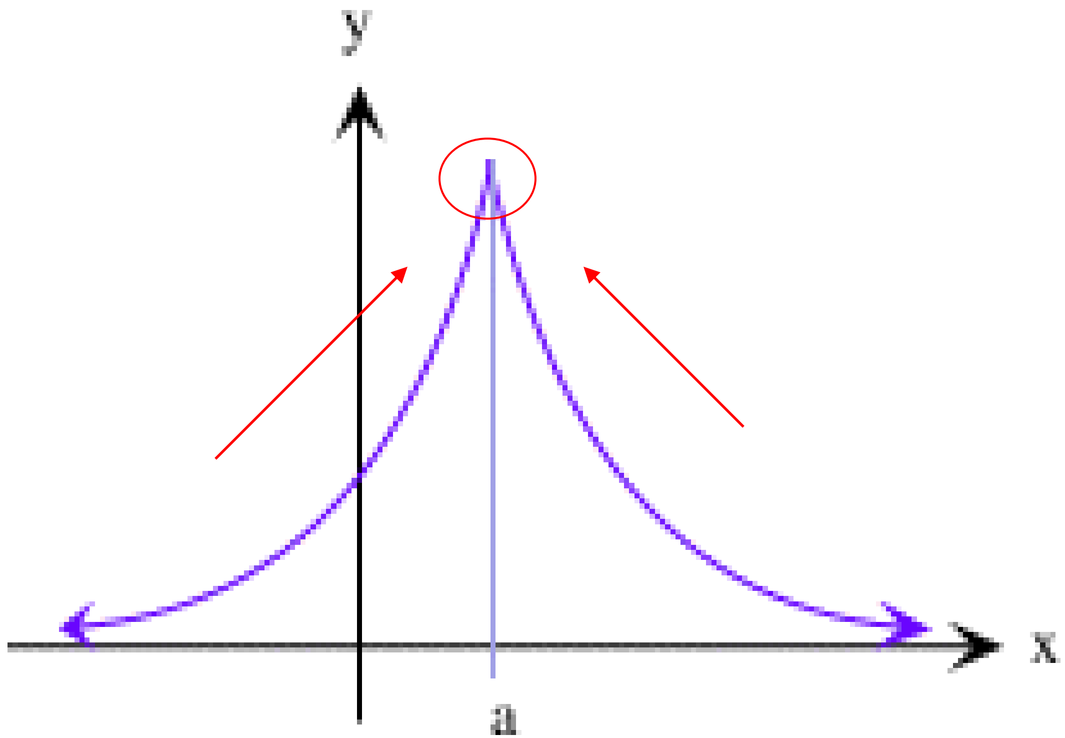
1. The limit does not exist, i.e.

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

does not exist.

This situation, when represented in graphical form leads to a cusp in the graph

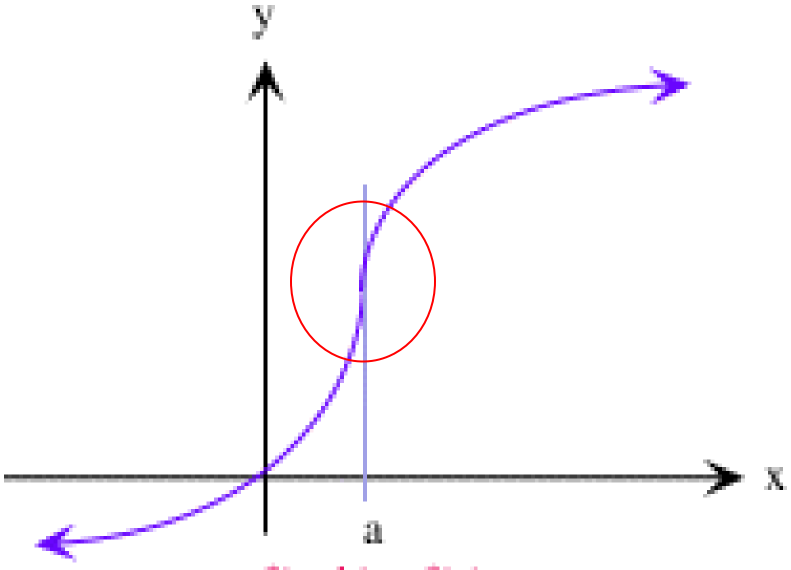
Undifferentiable



$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \text{ does not exist}$$

Undifferentiable

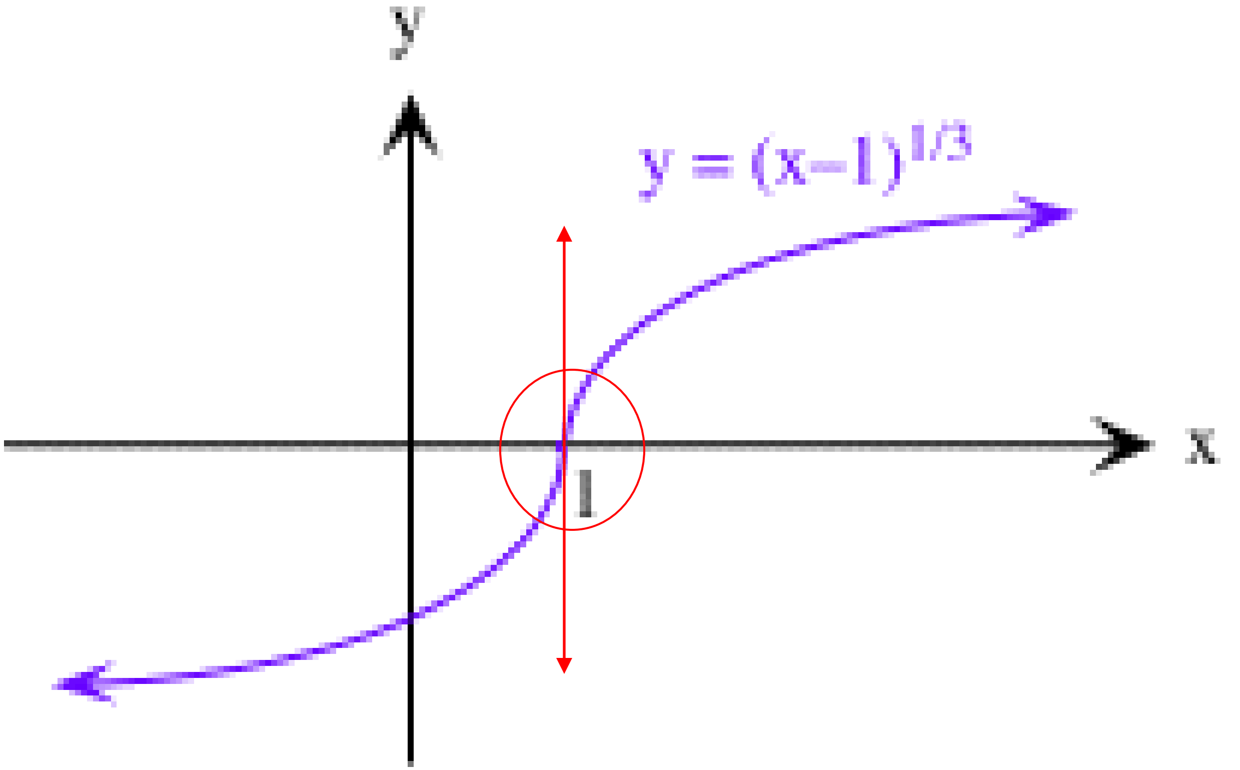
2. Limit goes to infinity, e.g. in the following case



$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \text{ is infinite}$$

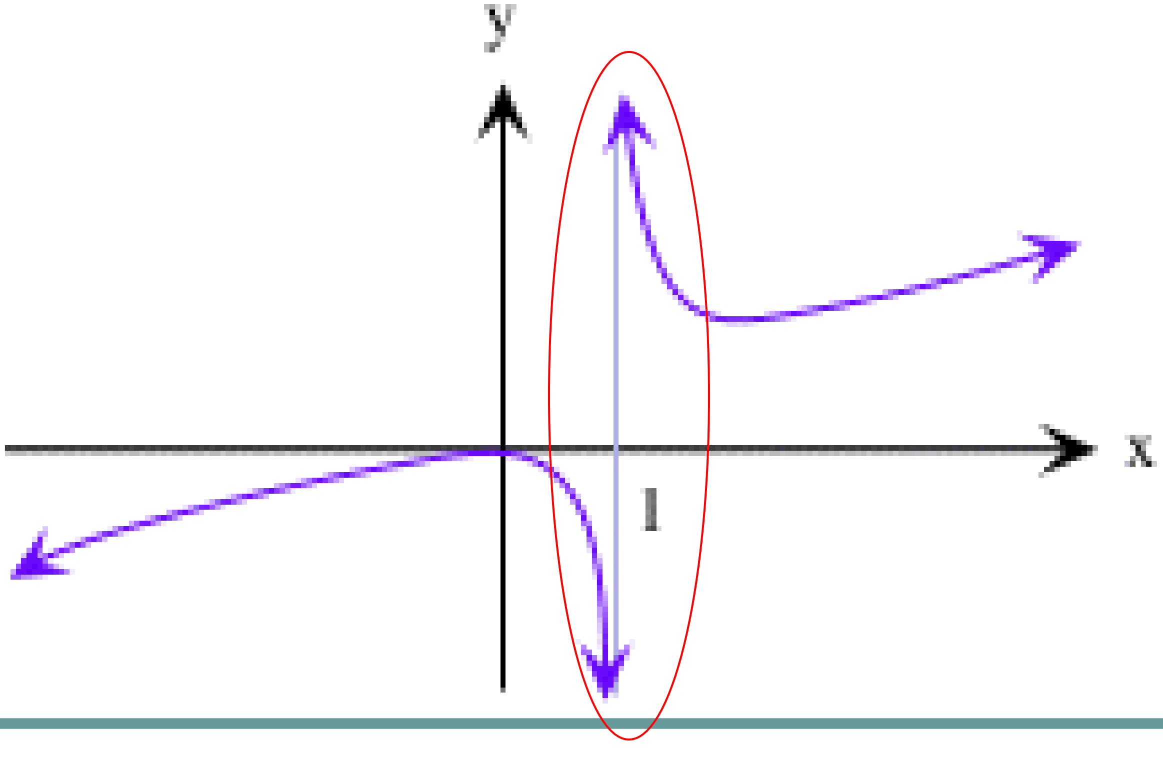
Isolated Non – Differentiable Points

- Consider the following examples



Isolated Non – Differentiable Points

$$f(x) = \frac{x^2}{x-1}$$



Difference of Opinion

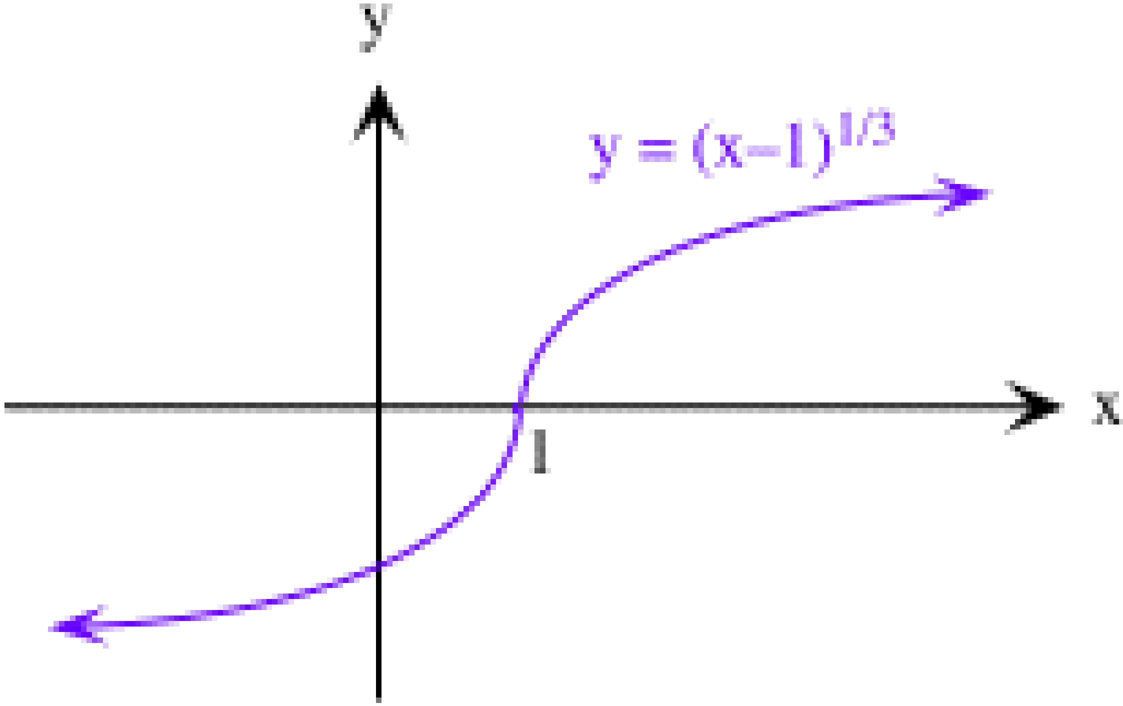
- f(x) is differentiable throughout its domain

Domain of f(x) is THE bone of contention

- f(x) is not differentiable throughout its domain

The Question

- Are all continuous functions differentiable ?



Another Question

- If f is not continuous at point a, then is it differentiable at that point ?

One More

- Are all differentiable functions continuous ?

YES

- Mathematically provable and easy to understand.

Proof of Continuity

- Suppose f(x) is differentiable at x=a

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$\lim_{h\to 0}[f(a+h)-f(a)]$$

$$= \lim_{h\to 0}\left[\frac{f(a+h)-f(a)}{h} * h\right]$$

$$= \lim_{h\to 0}\left[\frac{f(a+h)-f(a)}{h}\right] * \lim_{h\to 0} h$$

$$= f'(a) * 0$$

$$= 0$$

Continuity

$\lim_{x \to a} f(x)$ exists &

$\lim_{x \to a} f(x) = f(a)$