Lesson 2: The Constant, Power and Sum Rules

Example 1: Using the definition of derivative, determine the derivative of the following functions.

a) $f(x) = C x^0$ b) $f(x) = x$ c) $f(x) = x^2$ d) $f(x) = x^3$

a) $f'(x) = \lim_{h \to 0} \frac{C - C}{h}$

$= 0$

b) $f'(x) = \lim_{h \to 0} \frac{\cancel{x} + h - \cancel{x}}{h}$

$= \lim_{h \to 0} \frac{h}{h}$

$= 1$

c) $f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$

$= \lim_{h \to 0} \frac{\cancel{x^2} + 2x\cancel{h} + h^{\cancel{2}} - \cancel{x^2}}{\cancel{h}}$

$= 2x$

$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$

$= 3x^2$

Examine the above derivatives for a pattern.

Guess $f'(x)$ for $f(x) = x^4$

$f'(x) = 4x^3$

POWER RULE:

Given $f(x) = x^n$, then $f'(x) = nx^{n-1}$.

Example 2: Use limits to determine the derivative of g(x) = Cf(x)

$$g'(x) = \lim_{h \to 0} \frac{Cf(x+h) - Cf(x)}{h}$$

$$= \lim_{h \to 0} C\left[\frac{f(x+h) - f(x)}{h}\right]$$

$$= C \cdot \lim_{h \to 0}\left[\frac{f(x+h) - f(x)}{h}\right]$$

$$= C \cdot f'(x)$$

$g(x) = 4x^2$

$g'(x) = 8x$

Example 3: Differentiate using rules (not first principles).

a) $y = x^7$

$y' = 7x^6$

b) $y = 10x^5$

$y' = 50x^4$

c) $y = \frac{10}{x^5}$

$= 10x^{-5}$

$y' = -50x^{-6}$

d) $y = \sqrt{x}$

d) $y = x^{\frac{1}{2}}$

$y' = \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}}$

$= \frac{\sqrt{x}}{2x}$

Example 4: Use the definition of the derivative to prove that:
Given H(x) = f(x) + g(x), then H$'$(x) = f$'$(x) + g$'$(x)
Similarly; Given H(x) = f(x) - g(x), then H$'$(x) = f$'$(x) - g$'$(x)

$$H'(x) = \lim_{h\to 0} \frac{f(x+h)+g(x+h)-(f(x)+g(x))}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)+g(x+h)-g(x)}{h}$$

$$= \lim_{h\to 0} \left[\frac{f(x+h)-f(x)}{h} + \frac{g(x+h)-g(x)}{h}\right]$$

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} + \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$$

$$= f'(x) + g'(x)$$

Example 5: Differentiate.

a) f(x) = $x^2 - 5x + 6$

$$f'(x) = 2x - 5$$

b) f(x) = $10x^{-4} - 3x$

$$f'(x) = -40x^{-5} - 3$$

c) f(x) = $(x^3 - 4x)(2x + 1)$

$$= 2x^4 + x^3 - 8x^2 - 4x$$

$$f'(x) = 8x^3 + 3x^2 - 16x - 4$$

d) $\mathbf{y} = \dfrac{4x - 8\sqrt{x} + 14x^6}{2\sqrt{x}}$

$$y = \frac{4x^1}{2\sqrt{x}} - \frac{8\cancel{\sqrt{x}}}{2\cancel{\sqrt{x}}} + \frac{14x^6}{2\sqrt{x}}$$

$$= 2x^{\frac{1}{2}} - 4 + 7x^{\frac{11}{2}}$$

$$y' = 1x^{-\frac{1}{2}} + \frac{77}{2}x^{\frac{9}{2}}$$

$$= \frac{1}{\sqrt{x}} + \frac{77}{2}\sqrt{x^9}$$

Example 6: Find the equation(s) of the tangent(s) to $y = 5x^3$ with slope 60.

$y' = 15x^2$

$60 = 15x^2$

$\pm 2 = x$

$(2, 40)$

$y = mx + b$

$40 = 60(2) + b$

$-80 = b$

$\therefore \boxed{y = 60x - 80}$

$(-2, -40)$

$y = mx + b$

$80 = b$

$\therefore \boxed{y = 60x + 80}$