p.82

#3f) $s(t) = \dfrac{t^5 - 3t^2}{2t}$

$= \dfrac{t^5}{2t} - \dfrac{3t^2}{2t}$

$= \dfrac{1}{2}t^4 - \dfrac{3}{2}t$

$s'(t) = \dfrac{1}{2}(4)t^3 - \dfrac{3}{2}$

$= 2t^3 - \dfrac{3}{2}$

7c) $y = \dfrac{2}{x}$, $(-2, -1)$

$y = 2x^{-1}$

$y' = -2x^{-2}$

$y' = -2(-2)^{-2}$

$= -2\left(\dfrac{1}{-2}\right)^2$

$= -2\left(\dfrac{1}{4}\right)$

$= -\dfrac{1}{2}$

#10) $y = \frac{3}{x^2} - \frac{4}{x^3}$, $(-1, 7)$

* Normal → perpendicular line

$y = 3x^{-2} - 4x^{-3}$

$y' = -6x^{-3} + 12x^{-4}$

@ $x = -1$ $\quad y' = -6(-1)^{-3} + 12(-1)^{-4}$

$= 18$

$\therefore$ Slope of normal $= -\frac{1}{18}$ ✱

$y = mx + b$

$7 = -\frac{1}{18}(-1) + b$

$\frac{125}{18} = b$

$$\boxed{\therefore y = -\frac{1}{18}x + \frac{125}{18}}$$

#11) $y = \frac{3}{\sqrt[3]{x}}$ $\parallel$ $x + 16y + 3 = 0$

$y = 3x^{-1/3}$

$y' = -1x^{-4/3}$

$\left(-\frac{1}{16}\right)^3 = \left(-x^{-\cancel{4/3}}\right)^{\cancel{3}}$

$\left(-\frac{1}{4096}\right)^{-1/4} = \left(-x^{-4}\right)^{-1/4}$

$-8 = -x$

$\boxed{8 = x}$

$m = -\frac{A}{B}$

$= \boxed{-\frac{1}{16}}$

#18) $ax - 4y + 21 = 0$

$m = -\frac{A}{B}$

$= -\frac{a}{-4}$

$= \frac{a}{4}$

$y = \frac{a}{x^2}$, $x = -2$

@ $x = -2$

$y = \frac{a}{4}$

$= ax^{-2}$

$y' = -2ax^{-3}$

$= -2a(-2)^{-3}$

$= -2a\left(-\frac{1}{8}\right)$

$= \frac{a}{4}$

$ax - 4y + 21 = 0$

$a(-2) - 4\left(\frac{a}{4}\right) + 21 = 0$

$-2a - a + 21 = 0$

$-3a = -21$

$a = 7$

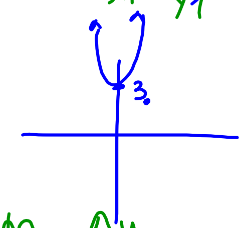
p.82 #7a) $y = 3x^4$, $(1,3)$

$$y' = 12x^3$$
$$= 12(1)^3$$
$$= 12$$

17a) $y = 2x^2 + 3$ $\quad (\underset{x_1}{2}, \underset{y_1}{3})$

$$y' = 4x$$

$P(x, \underbrace{2x^2+3}_{y})$



$$m = \frac{\Delta y}{\Delta x}$$

$$4x = \frac{3 - (2x^2+3)}{2 - x}$$

$$8x - 4x^2 = -2x^2$$
$$0 = 2x^2 - 8x$$
$$0 = 2x(x-4)$$
$$x = 0, \; 4$$
$$y = 3, \; 35$$

* We can find eqn of tangents

① @ $(0,3)$

$$m = 4(0)$$
$$= 0$$

$$y = mx + b$$
$$3 = 0(0) + b$$
$$3 = b$$

$$\boxed{\therefore y = 3}$$

② $(4,35)$

$$m = 4(4)$$
$$= 16$$

$$y = mx + b$$
$$35 = 16(4) + b$$
$$-29 = b$$

$$\boxed{\therefore y = 16x - 29}$$

#11) $x + 16y + 3 = 0$

$m = -\frac{A}{B}$

$= -\frac{1}{16}$

$$y = \frac{3}{\sqrt[3]{x}}$$

$$= 3x^{-1/3}$$

$$y' = -1x^{-4/3}$$

$$\left(-x^{-4/3}\right)^{\cancel{3}} = \left(-\frac{1}{16}\right)^{3}$$

$$\left(\cancel{-}x^{\cancel{-4}}\right)^{-1/4} = \left(\cancel{-}\frac{1}{4096}\right)^{-1/4}$$

$$x = \left(\frac{1}{4096}\right)^{-1/4}$$

$$= \pm 8$$

Lesson 3: The Product Rule and the Power of a Function Rule

Warm Up: **Differentiate** a) $y = (x^2 - 2)(x^4 + 3)$ b) $y = 2x^3 (x^2 - 4)$

$= x^6 + 3x^2 - 2x^4 - 6$

$y' = 6x^5 - 8x^3 + 6x$

b) $y = 2x^5 - 8x^3$

$y' = 10x^4 - 24x^2$

$y = (x + 3x^2 - 7x^3 + x^4)(x^3 + 2x - 7)$

Example 0: Show with a counter example that when $f(x) = \boxed{g(x)}h(x)$, $f'(x) \neq g'(x)h'(x)$

$$f(x) = g(x) \cdot H(x)$$

$$f'(x) = \lim_{h \to 0} \frac{g(x+h) \cdot H(x+h) - g(x) \cdot H(x)}{h}$$

$$= \lim_{h \to 0} \frac{g(x+h)H(x+h) - g(x) \cdot H(x+h) + g(x)H(x+h) - g(x) \cdot H(x)}{h}$$

$$= \lim_{h \to 0} \frac{H(x+h)\left[g(x+h) - g(x)\right] + g(x)\left[H(x+h) - H(x)\right]}{h}$$

$$= \lim_{h \to 0} H(x+h)\frac{\left[g(x+h) - g(x)\right]}{h} + \lim_{h \to 0} g(x)\frac{\left[H(x+h) - H(x)\right]}{h}$$

$$= H(x+\cancel{h}) \cdot g'(x) + g(x) \cdot H'(x)$$

$h=0$

Example 2: Differentiate. DO NOT SIMPLIFY

a) $y = (2x + 6)(3x + 2)$

$y' = 2(3x+2) + (2x+6)(3)$

b) $y = 5x^5(4x^4 - 3x)$

$y' = 25x^4(4x^4 - 3x) + 5x^5(16x^3 - 3)$

c) $y = \sqrt{x}(5x^3 - 4x + 6)$

$y' = \frac{1}{2}x^{-\frac{1}{2}}(5x^3 - 4x + 6) + \sqrt{x}(15x^2 - 4)$

Product Rule

$$f(x) = g(x) \cdot h(x) \cdot p(x)$$

$$f'(x) = g'(x) \cdot h(x)p(x) + g(x) \cdot h'(x)p(x)$$

$$+ g(x) \cdot h(x) \cdot p'(x)$$

Example 3: Differentiate. Do **Not** Simplify.

a) $y = (x^5 - 10x)^3 + 8x^2$ b) $y = \sqrt[3]{5x^2 - 7x + 3}$ $y = \dfrac{10}{(2x^2 - 9)^4}$

a) $y' = 3(x^5 - 10x)^2 \cdot (5x^4 - 10) + 16x$

b) $y = (5x^2 - 7x + 3)^{\frac{1}{3}}$

$y' = \frac{1}{3}(5x^2 - 7x + 3)^{-\frac{2}{3}} \cdot (10x - 7)$

c) $y = 10(2x^2 - 9)^{-4}$

$y' = -40(2x^2 - 9)^{-5} \cdot (4x)$