Lesson 4: The Quotient Rule

The Quotient Rule:

Given $f(x)=\dfrac{g(x)}{h(x)}$, then $f(x)=\dfrac{h(x)g'(x)-g(x)h'(x)}{[h(x)]^2}$

$$f(x)=g(x)\cdot[h(x)]^{-1}$$

$$f'(x)=g'(x)[h(x)]^{-1}+g(x)(-1)[h(x)]^{-2}h'(x)$$

$$=\frac{g'(x)}{h(x)}\cdot\frac{h(x)}{h(x)}-\frac{g(x)h'(x)}{[h(x)]^2}$$

$$=\frac{g'(x)\cdot h(x)}{[h(x)]^2}-\frac{g(x)\cdot h'(x)}{[h(x)]^2}$$

$$=\frac{g'(x)\cdot h(x)-g(x)\cdot h'(x)}{[h(x)]^2}$$

Example 1: Differentiate. Do NOT simplify.

a) $f(x)=\dfrac{x^2-6x+3}{x^3-4x+1}$

$$f'(x)=\frac{(2x-6)(x^3-4x+1)-(3x^2-4)(x^2-6x+3)}{[x^3-4x+1]^2}$$

b) $f(x)=\dfrac{\sqrt{x}-3x+6}{5x^8-6x}$

$$y'=\frac{(\frac{1}{2}x^{-\frac{1}{2}}-3)(5x^8-6x)-(40x^7-6)(\sqrt{x}-3x+6)}{(5x^8-6x)^2}$$

c) $f(x)=\dfrac{(2x^2-3x)(5x-2)}{(4x^3-3)}$

$$f'(x)=\frac{(4x^3-3)[(4x-3)(5x-2)+(2x^2-3x)(5)]-(2x^2-3x)(5x-2)(12x^2)}{(4x^3-3)^2}$$