9. From "First Principles" show that if $\mathbf{f}(x) = \dfrac{1}{x^2-3}$, then $\mathbf{f'}(x) = \dfrac{-2x}{(x^2-3)^2}$ [4]

$$f'(x) = \lim_{h \to 0} \frac{\dfrac{1}{(x+h)^2-3} - \dfrac{1}{x^2-3}}{h}$$

$$= \lim_{h \to 0} \frac{x^2-3-\left[(x+h)^2-3\right]}{\left[(x+h)^2-3\right](x^2-3)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2}-\cancel{3}-\left(\cancel{x^2}+2x\cancel{h}+h^{\cancel{2}\,1}\cancel{-3}\right)}{\left(\qquad\right)} \cdot \frac{1}{\cancel{h}}$$

$$= \lim_{h \to 0} \frac{-2x-h}{\left[(x+h)^2-3\right]\left[x^2-3\right]}$$

$$= \frac{-2x}{(x^2-3)(x^2-3)}$$

$$= \frac{-2x}{(x^2-3)^2}$$

3. Find the equation(s) of the tangent(s) to $y = 2x^3 - 16$ at the point(s) where the curve touches the x-axis. [5]

$y = 2x^3 - 16$

$0 = 2(x^3 - 8)$

$= 2(x-2)(x^2+2x+4)$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 0 & -8 \\ & & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 0R \end{array}$$

$x = 2$

$y = 0$

$(2, 0)$

$b^2 - 4ac$

$4 - 4(1)(4) < 0$

< 0 no zeros

$= 0$ 1 zero

> 0 2 zeros

$y' = 6x^2$

$y' = 6(2)^2$

$= 6(4)$

$= 24$

$\therefore y = mx + b$

$0 = 24(2) + b$

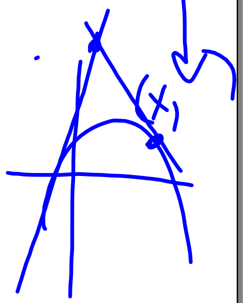
$-48 = b$

$\boxed{\therefore y = 24x - 48}$

1. Determine the equation(s) of the tangent(s) to the curve $y = 10x - x^2 - 16$ from (1, 18).

2. The line $y + 2x = 0$ is tangent to $y = F(x)$. Determine $F(x)$ if $F'(x) = 4x^3 + 2$.

① (1, 18) not on y.



$$y' = \boxed{10 - 2x}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$10 - 2x = \frac{18 - (10x - x^2 - 16)}{1 - x}$$

$$(10 - 2x)(1 - x) = 18 - 10x + x^2 + 16$$

$$10 - 10x - 2x + 2x^2 = 18 - 10x + x^2 + 16$$

$$x^2 - 2x - 24 = 0$$

$$(x - 6)(x + 4) = 0$$

$$x = 6, \quad x = -4$$

$$y = 8, \quad y = -72$$

$$m = 10 - 2(6) = -2, \quad m = 10 - 2(-4) = 18$$

$y = mx + b$

$8 = -2(6) + b$

$20 = b$

$\therefore y = -2x + 20$

$y = mx + b$

$-72 = 18(-4) + b$

$0 = b$

$y = 18x$

2. The line $y + 2x = 0$ is tangent to $y = F(x)$. Determine $F(x)$ if $F'(x) = \underbrace{4x^3 + 2}$.

$y = -2x$ } eqn of tangent

$m = -2$

$4x^3 + 2 = -2$

$4x^3 = -4$

$x^3 = -1$

$x = -1$ * * *

$y = -2(-1)$
$= 2$

$F(x) = x^4 + 2x + \underline{\underline{c}}$

$2 = (-1)^4 + 2(-1) + c$

$2 = -1 + c$

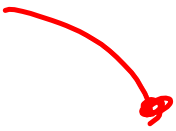
$3 = c$

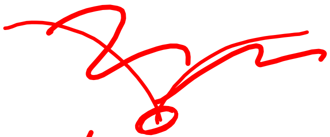
$\therefore \boxed{F(x) = x^4 + 2x + 3}$

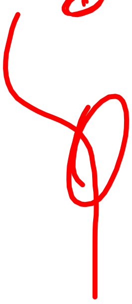
1. Each of the following functions is defined at x = 3 but not differentiable at x = 3. Cleary explain why each function is not differentiable there.

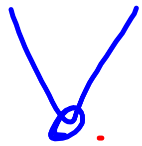
a) $f(x)=\sqrt{3-x}$ b) $f(x)=(x-3)^{\frac{1}{3}}$ c) $f(x)=|x-3|$

$\sqrt[3]{x-3}$









$$(((x^2+3x+1)^2+1)^5+5)^8$$

$$= 8(((x^2+3x+1)^2+1)^5+5)^7 \cdot 5((x^2+3x+1)^2+1)^4$$

$$\cdot\, 2(x^2+3x+1)(2x+3)$$

$$y = \frac{(2x^2+4x)^4}{14x}$$

$$y' = \frac{\boxed{(14x)\,4(2x^2+4x)^3(4x+4) - (2x^2+4x)^4(14)}}{(14x)^2}$$

$$y = \frac{(12x^3 - 3x(4-8x^3)^5)^6}{(8x^2-5x)^3}$$

$$y' = \frac{6(12x^3-3x(4-8x^3)^5)^5 \cdot (36x^2 - [3(4-8x^3)^5 + 3x(5)(4-8x^3)^4(-24x^2)] \cdot (8x^2-5x)^3 - (\text{top}) \cdot 3(8x^2-5x)^2(16x-5)}{(8x^2-5x)^6}$$