Unit 2: The Derivative

Lesson 1: The Derivative

Warm Up: Using the Limit method of Unit 1; find the slope of the tangent (i.e. the instantaneous rate of change) to y $=x^2$ at the points:

a) x = 1 b) x = -2 c) x = 6

$y=(1)^2$
$=1$
$y=(1+h)^2$
$=h^2+2h+1$

$\lim_{h\to 0}\frac{(h^2+2\cancel{h}+\cancel{1})-(\cancel{1})}{\cancel{h}}$

$=2$

IDEA!!! Why not do this once, since all the calculations are "parallel".

The Derivative Function $f'(x)$.

The Function Whose **Output** is the **Instantaneous Rate of Change** (i.e. the Slopes of the Tangents)

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Graphs

Example 2:
a) Determine the derivative of $y = x^2$.
b) Calculate the slopes of the tangents at $x = 1$, $x = -2$, $x = 6$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$$
$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2x\cancel{h} + h^{\cancel{2}} - \cancel{x^2}}{\cancel{h}}$$
$$= \lim_{h\to 0} 2x + h$$
$$= 2x$$

$\therefore$ $f'(1) = 2(1) = 2$ $\quad f'(-2) = 2(-2) = -4$

$f'(6) = 2(6) = 12$

Example 3:
a) Determine the equation of the tangent to $y = x^3$ at $x = 1$.
b) Locate the second place where the tangent intersects the curve.

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$$
$$= \lim_{h\to 0} \frac{(\cancel{x^3} + 3x^2\cancel{h} + 3xh^{\cancel{2}\,1} + h^{\cancel{3}\,2}) - \cancel{x^3}}{\cancel{h}}$$
$$= \lim_{h\to 0} 3x^2 + 3xh + h^2$$
$$= 3x^2$$

$f'(1) = 3(1)^2 = 3$

$y = mx + b$
$1 = 3(1) + b$
$-2 = b$

$\therefore y = 3x - 2$

b) POI
$x^3 = 3x - 2$
$x^3 - 3x + 2 = 0$
$(x-1)(x^2+x-2) = 0$
$(x-1)(x-1)(x+2) = 0$
$(x-1)^2(x+2) = 0$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0\text{R} \end{array}$$

$\therefore$ The tangent intersects $y = x^3$ @ $x = -2$

Example 4: A ball is thrown upward from a balcony 10m above the ground. The height, in meters, is given by the function t is in seconds.

$h(t) = -4.9t^2 + 29.4t + 10$

a) Determine the velocity of the ball after 1s.
b) What is the velocity of the ball at 3 seconds?
c) What is the maximum height the ball reaches?
d) What is the total distance the ball travels in the first 5 seconds?

$$h'(t) = \lim_{h\to 0} \frac{-4.9(t+h)^2 + 29.4(t+h) + 10 - (-4.9t^2 + 29.4t + 10)}{h}$$

$$= \lim_{h\to 0} \frac{-4.9(t^2 + 2th + h^2) + 29.4t + 29.4h + \cancel{10} + 4.9t^2 - 29.4t - \cancel{10}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{-4.9t^2} - 9.8th - 4.9h^2 + \cancel{29.4t} + 29.4h + \cancel{4.9t^2} - \cancel{29.4t}}{h}$$

$$= \lim_{h\to 0} \frac{-9.8t\cancel{h} - 4.9h^{\cancel{2}} + 29.4\cancel{h}}{\cancel{h}}$$

$$= -9.8t + 29.4$$

a) $v(t) = h'(t) = -9.8t + 29.4$

$v(1) = 19.6\,m/s$

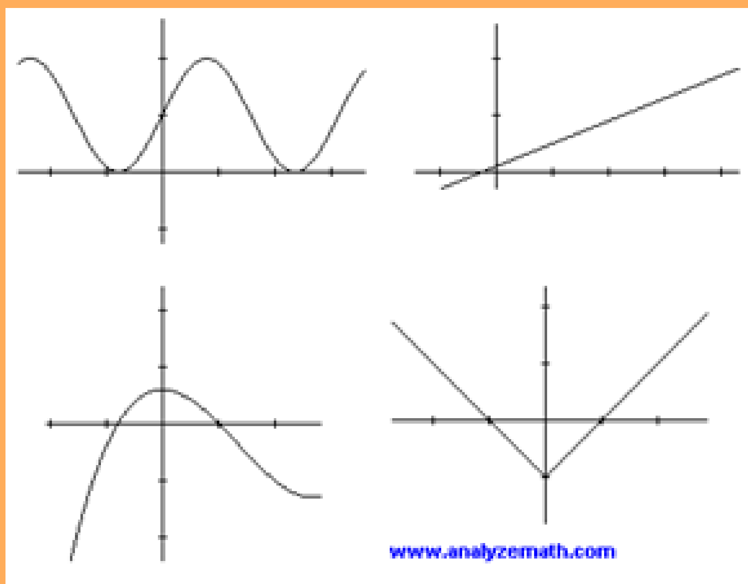
b) $v(3) = 0\,m/s$

c) $h(3) = -4.9(3)^2 + 29.4(3) + 10$

$= 54.1\,m$

d) $h(5) =$

Example 5: Given y = f(x), sketch the graph of y = f′(x).



Attachments

Derivative Plot B.GSP

Derivative Plot.GSP