Lesson 3: Developing the Derivative of the Sine Function, Partners Activity. No Help

Numerical Tools

Use tables of values to approximate the two limits below. (IN RADIANS!!)

h	$\frac{\sin(h)}{h}$
1	0.84
0.1	0.99
0.001	0.999
-1	0.84
-0.1	0.99
-0.001	0.999

h	$\frac{\cos(h)-1}{h}$
1	-0.459
0.1	-0.04
0.001	-4.9×10^{-4}
-1	0.459
-0.1	0.0499
-0.001	4.9×10^{-4}

$$\lim_{h\to0}\frac{\sin(h)}{h} = \underline{1}$$

$$\lim_{h\to0}\frac{\cos(h)-1}{h} = \underline{0}$$

**The first of these can be proven rigorously with a geometric construction and then used to prove the second. We are not responsible for that in this course.

Lesson 3: Developing the Derivative of the Sine Function

Key Formula

Recall the addition formula for sine from MHF.

sin(x + h) = $\sin x \cos h + \sin h \cos x$

$$\cos(x+h) = \cos x \cos h - \sin x \sin h$$

Algebraic Proof from First Principles

In your notes; evaluate the derivative of the sine function using the limit definition.
Hint: You will use results from Step I and the formula from Step II in the simplification.

$$f'(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x(\cos h - 1) + \cos x \sin h}{h}$$

$$= \lim_{h \to 0} \sin x \left[\frac{\cos h - 1}{h}\right] + \lim_{h \to 0} \cos x \left[\frac{\sin h}{h}\right]$$

$$= \sin x (0) + \cos x \cdot 1$$

$$= \cos x$$

Other Trig Derivatives

Use your main result If y = sin(x), then y$'$ = $\underline{\cos x}$ to evaluate the derivatives below.

1. Rewrite y = cos(x) in terms of the sine function. Differentiate this expression to determine the derivative of the cosine function. Show all work.

$$f'(x) = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\cos x(\cos h - 1) - \sin x \sin h}{h}$$

$$= \lim_{h \to 0} \left[\cos x \left(\frac{\cos h - 1}{h}\right)\right] - \lim_{h \to 0} \left[\sin x \left(\frac{\sin h}{h}\right)\right]$$

$$= \cos x \cdot 0 - \sin x \cdot 1$$

$$= -\sin x$$

2. Differentiate y = tan(x).

$$= \frac{\sin x}{\cos x}$$

$$\frac{d \sin x}{dx} = \cos x$$

$$\frac{d \cos x}{dx} = -\sin x$$

$$y' = \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{(\cos x)^2}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

4. Differentiate. Do Not Simplify.

a) $y = \dfrac{x - \sin x}{x + \cos x}$

$$y' = \frac{(1-\cos x)(x+\cos x) - (x-\sin x)(1-\sin x)}{(x+\cos x)^2}$$

b) $y = \tan x \sqrt{\cos x + e^x}$

$$y' = (\sec x)^2 (\cos x + e^x)^{\frac{1}{2}} +$$

$$(\tan x)\tfrac{1}{2}(\cos x + e^x)^{-\frac{1}{2}}(-\sin x + (e^x)(\ln e)(1))$$

c) $y = \ln^5(\cos^2 x)$ $= [\ln(\cos^2 x)]^5$

$$5(\ln(\cos^2 x))^4 \cdot \frac{1}{\cos^2 x} \cdot 2(\cos x) \cdot -\sin x$$

5. The position of a certain oscillating (vibrating) object is given by

$s(t) = 8\cos(2t)$.

a. Determine the velocity equation for the object.

b. What is the initial velocity of the object?

c. When is the object at rest (answer for $0 < t < 10$s)?

Attachments

Trig Derivatives.gsp