Lesson 1: Derivative of Exponential Functions

Part A

Part A: Graphical Evidence--Partners

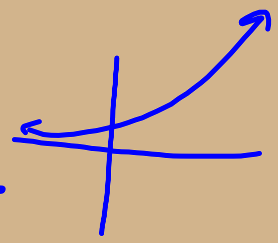
6. Describe the derivative of $y = A^x$.

↳ exponential

↳ can be below $y=A^x$ $\{x=2\}$

↳ can be above $y=A^x$ $\{x=5\}$

↳ can be a reflection in x $\{A=\frac{1}{2}\}$



i) The derivative of an exponential function appears to be ____________

A) Polynomial B) Rational C) Trigonometric D) Exponential E) Linear

ii) The derivative of $y = A^x$ is an exponential of the form ____________

A) $y = r^x$ B) $y = C(r)^x$ C) $y = r^x + d$ D) $y = C(r)^x + d$ E) It's not exponential

Part B: An Important Property of the Exponential Derivative

Part B

* Same Base

$$\frac{C \cancel{f(x)}}{\cancel{f(x)}}$$

iii) For the exponential function, $f(x) = A^x$, the ratio $\frac{f'(x)}{f(x)}$ is C
Some constant

A) Zero B) Constant C) Linear D) Exponential E) Not Predictable

iv) So, when $y = A^x$ then ______________

A) $y' = A^x$ B) $y' = k(A)^x$ C) $y' = B^x$ D) $y' = kB^x$

v) For what value of A does the ratio appear to be one? ______________

Part C: An Algebraic Approach to Finding the Derivative

$$y' = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$ } definition of the derivative

$$= \lim_{h\to 0}\frac{A^{x+h}-A^{x}}{h}$$ } Sub $f(x)=A^x$, $f(x+h)=A^{x+h}$

$$= \lim_{h\to 0}\frac{A^{x}(A^{h}-1)}{h}$$ } common factored A^x

$$= A^{x}\lim_{h\to 0}\frac{(A^{h}-1)}{h}$$ } Since A^x does not depend on h, take it outside the limit.

$$= k_A A^x$$ } Call $\boxed{k_A = \lim_{h\to 0}\frac{A^h - 1}{h}}$

vi) Which choice correctly completes the sentence?

"*When f(x) = A^x, then f′(x) = $k_A A^x$, where k_A* ______________"

A) is a constant B) is the value of a limit C) $= \lim_{h\to 0}\frac{A^h - 1}{h}$ D) depends on A E) Any of these.

Part D: Approximating the Limit to Find k_A

Part D

Name k_A	Expression	Value
k2 2	$\lim_{h\to 0}\frac{2^h-1}{h}$	$k_2 \doteq 0.693$
k3 3	$\lim_{h\to 0}\frac{3^h-1}{h}$	$k_3 \doteq 1.0986$
k4.232	$\lim_{h\to 0}\frac{4.232^h-1}{h}$	$k_{4.232} \doteq 1.443$
k0.5 0.5	$\lim_{h\to 0}\frac{0.5^h-1}{h}$	$k_{0.5} \doteq -0.6931$
k0.25	$\lim_{h\to 0}\frac{0.25^h-1}{h}$	$k_{0.25} \doteq -1.386$
k2.7182	$\lim_{h\to 0}\frac{2.7182^h-1}{h}$	$k_{2.7182} \doteq 1.0000$ **

vi) The value of this limit is greater than one when A = 3, 4.232

vii) The value of this limit is less than one when A = 2, ½, ¼

viii) The value of this limit is near one when A = 2.7182 = e

ix) Use the limits from above to help differentiate the following functions:

a) $y = 2^x$ b) $y = 4.232^x$ c) $y = 0.5^x$ d) $y = 2.718282^x$

a) $y' = k_2(2^x)$
$= 0.693(2^x)$

b) $y' = k_{4.232}(4.232^x)$
$= 1.443(4.232^x)$

d) $y' = k_{2.718282}(2.718282^x)$
$= 2.718282^x$
$y' = y$

General Rule

if $f(x) = b^x$

then $f'(x) = \ln b(b^x)$

ex: $f(x) = 5^x$

$f'(x) = \ln 5 \cdot 5^x$

ex: $f(x) = 7^x$

$f'(x) = \ln 7 \cdot 7^x$

ex: $f(x) = 3^{4x}$

$f'(x) = \ln 3 \cdot 3^{4x} \cdot 4$

if $f(x) = b^{g(x)}$

then $f'(x) = \ln b \cdot b^{g(x)} \cdot g'(x)$

Examples:

1a) $f(x)=2^{3x}$

$f'(x)=\ln 2\cdot 2^{3x}\cdot 3$

b) $f(x)=2.7^{x}+x^{4}$

$f'(x)=\ln 2.7\cdot 2.7^{x}+4x^{3}$

c) $f(x)=6^{6t-2t^{2}}$

$f'(x)=6^{6t-2t^{2}}\cdot \ln 6\cdot (6-4t)$

d) $f(x)=x^{4}\cdot 5^{x}$

$f'(x)=4x^{3}(5^{x})+(5^{x}\cdot \ln 5(x^{4}))$

e) $h(t)=2^{\frac{t^{3}-t^{2}}{t^{5}}}$

$h'(t)=2^{\frac{t^{3}-t^{2}}{t^{5}}}\cdot \ln 2\cdot \left[\frac{(3t^{2}-2t)(t^{5})-(t^{3}-t^{2})(5t^{4})}{[t^{5}]^{2}}\right]$

Example: $f(t)=14^{5t-2}\cdot e^{4t^2}$

$$f'(t)=14^{5t-2}\cdot \ln 14\cdot 5\cdot e^{4t^2}+14^{5t-2}\cdot e^{4t^2}\cdot \cancel{\ln e}^{\,1}\cdot 8t$$

$f'(t)=0$

@ $t=?$

$$0=\boxed{14^{5t-2}}\cdot \ln 14\cdot 5\cdot \boxed{e^{4t^2}}+\boxed{14^{5t-2}}\cdot \boxed{e^{4t^2}}\cdot 8t$$

$$0=14^{5t-2}\cdot e^{4t^2}\left[5\ln 14+8t\right]$$

$14^{5t-2}=0$ $\quad$ $e^{4t^2}=0$

DNE

$$5\ln 14+8t=0$$

$$\frac{8t}{8}=-\frac{5\ln 14}{8}$$

$$t\doteq -1.65$$