

Lesson 3: Optimization Problems**Example 1:** (Polynomial)

A box with an open top is to be constructed from a square piece of cardboard, 3m wide, by cutting out a square piece from each of the four corners and bending up the sides. Find the largest volume of such a box.

$$V = lwh$$

$$= (3-2x)(3-2x)(x)$$

$$= 9x - 12x^2 + 4x^3$$

$$V'(x) = 9 - 24x + 12x^2$$

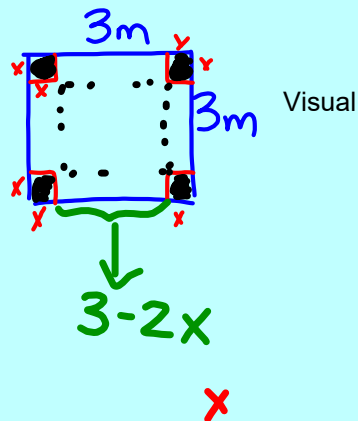
$$12x^2 - 24x + 9 = 0$$

$$(2x-1)(2x-3) = 0$$

$$x = \frac{1}{2} \quad \cancel{\frac{3}{2}} \quad \text{too BIG}$$


$$V\left(\frac{1}{2}\right) = \left(3 - 2\left(\frac{1}{2}\right)\right)^2 \left(\frac{1}{2}\right)$$

$$= 2\text{m}^3$$



Example 2: (Polynomial)

A string of length 50 m is cut in two pieces. One piece is bent into a circle and the other is bent into an equilateral triangle. Find an equation for the total area of the two shapes, and where should the string be cut to minimize the area?



$$x + y = 50 - x$$

$$h = \sqrt{\left(\frac{x}{2}\right)^2 - \left(\frac{x}{6}\right)^2}$$

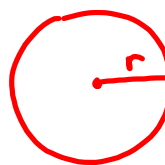
$$= \sqrt{\frac{x^2}{4} - \frac{x^2}{36}}$$

$$= \sqrt{\frac{3x^2}{36}} = \frac{x\sqrt{3}}{6}$$

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}\left(\frac{x}{3}\right)\left(\frac{x\sqrt{3}}{6}\right)$$

$$= \frac{x^2\sqrt{3}}{36}$$


 Visual

$$C = 2\pi r$$

$$50 - x = 2\pi r$$

$$\frac{50 - x}{2\pi} = r$$

$$A = \pi r^2$$

$$= \pi \left(\frac{50 - x}{2\pi}\right)^2$$

$$\text{Total Area} = \frac{x^2\sqrt{3}}{36} + \frac{(50 - x)^2}{4\pi}$$

$$A' = \frac{2x\sqrt{3}}{36} + \frac{2(50 - x)(-1)}{4\pi}$$

$$0 = \frac{2x\sqrt{3}}{36} \cdot \frac{\pi}{\pi} + \frac{(-100 + 2x)}{4\pi} \cdot \frac{\pi}{\pi}$$

$$0 = \frac{2\pi x\sqrt{3}}{36\pi} - 900 + 18x$$

$$900 = 2\pi x\sqrt{3} + 18x$$

$$900 = x(2\sqrt{3}\pi + 18)$$

$$31.2 = x$$

We need to cut the string at 31.2 m to minimize the area.

Example 3: (Economics)

A manufacturer of calculators produces x calculators per day at a daily cost in dollars of $C(x) = 40x - 0.035x^2 + 1250$. If the calculators are sold for $60 - 0.05x$ each, find the value of x that maximizes the daily profit. What is the price for maximum profit?

$$P = R - C$$

$$R = (60 - 0.05x)x$$

$$P = (60 - 0.05x)x - (40x - 0.035x^2 + 1250)$$

$$= 0.015x^2 + 20x - 1250$$

$$P'(x) = -0.03x + 20$$

$$0 = -0.03x + 20$$

$$667 = x$$

$$\begin{aligned} \text{price} &= 60 - 0.05(667) \\ &= \$26.65 \end{aligned}$$

Example 4: (Rational)

An open topped storage box with a square base is to have a capacity of 5m^3 . Material for the sides costs $\$1.60/\text{m}^2$, while that for the bottom costs $\$2.00/\text{m}^2$. Find the dimensions that will minimize the cost of the material. What is the minimum cost?

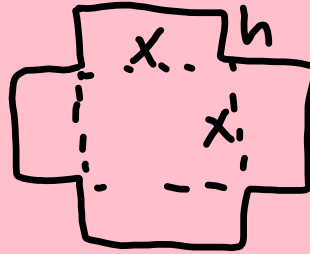
$$V = 5\text{m}^3 \quad SA = 4xh + x^2$$

$$V = lwh \quad SA = 4x \cdot 5 + x^2$$

$$5 = x^2 \cdot h$$

$$\frac{5}{x^2} = h$$

$$= \frac{20}{x} + x^2$$



$$C(x) = 1.60 \left(\frac{20}{x} \right) + 2(x^2)$$

$$= \frac{32}{x} + 2x^2$$

$$C'(x) = -\frac{32}{x^2} + 4x$$

$$0 = -\frac{32}{x^2} + 4x$$

$$= \frac{-32 + 4x^3}{x^2}$$

$$0 = -32 + 4x^3$$

$$+32 = 4x^3$$

$$+8 = x^3$$

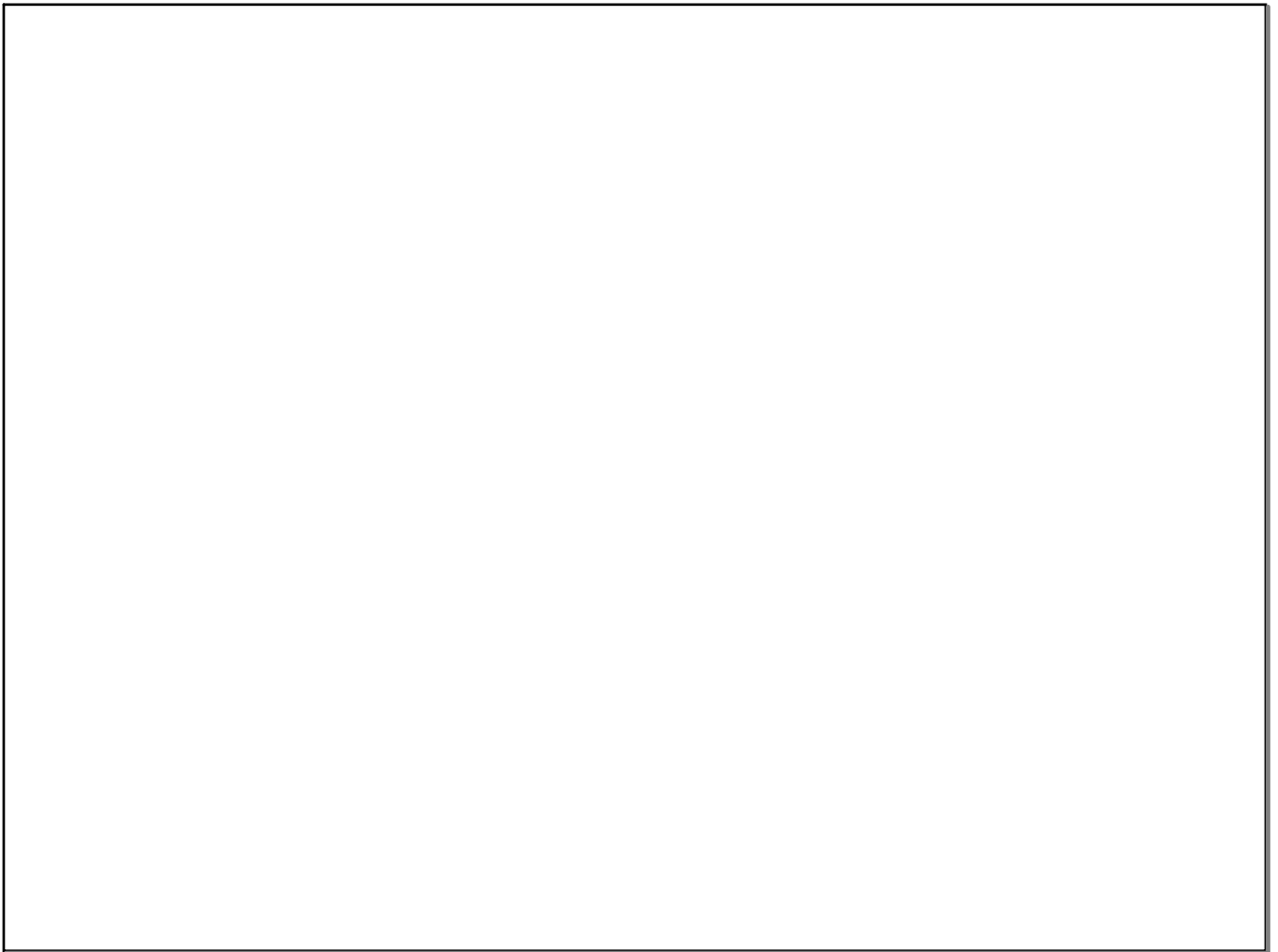
$$C(2) = \$24$$

p.135 #10)

$$30 \leq x \leq 120$$

$$r(x) = \frac{1}{4} \left(\frac{4900}{x} + x \right)$$

answer should be \$80.50
(Book is wrong)



Attachments

Ex2TriangleCircle.gsp