Lesson 3: Optimization Problems

Example 5: (Composite Functions, Distances)
Find the point on the parabola $2y = x^2$ that is closest to (-4, 1).

$y = \frac{x^2}{2}$ $\left(x, \frac{x^2}{2}\right)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-4 - x)^2 + \left(1 - \frac{x^2}{2}\right)^2}$$

$$= \sqrt{16 + 8x + x^2 + 1 - x^2 + \frac{x^4}{4}}$$

$$= \sqrt{17 + 8x + \frac{x^4}{4}}$$

$$d' = \frac{1}{2}\left(17 + 8x + \frac{x^4}{4}\right)^{-\frac{1}{2}} \cdot (8 + x^3)$$

$$0 = \text{～～～～～～}$$

$$0 = 8 + x^3$$

$$-8 = x^3$$

$$-2 = x \qquad y = \frac{(-2)^2}{2} = 2$$

∴ The point closest is (-2, 2)

Example 6: (Composite Functions) -See Lesson 3 computer
Sam lives across the river and 180 m downstream her boyfriend Paul. If Sam swims at 4 m/s and runs at 5 m/s what is the quickest she can meet him? The river is 75 m wide.

→ time

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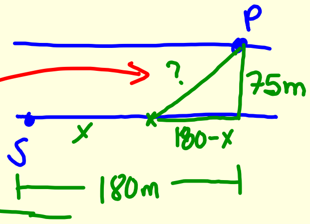
$V_s = 4 m/s$
$V_r = 5 m/s$



$? = \sqrt{75^2 + (180-x)^2}$

$= \sqrt{5625 + 32400 - 360x + x^2}$

$= \sqrt{38025 - 360x + x^2}$

$t_R = \frac{x}{5}$ $t_s = \frac{\sqrt{38025-360x+x^2}}{4}$

$T_t = t_R + t_s$

$T = \frac{x}{5} + \frac{\sqrt{38025-360x+x^2}}{4}$

$T' = \frac{1}{5} + \frac{1}{4}\left(\frac{1}{2}\right)(38025-360x+x^2)^{-1/2} \cdot (-360+2x)$

$= \frac{1}{5} + \frac{(-360+2x)}{8\sqrt{38025-360x+x^2}}$

$= \frac{8\sqrt{38025-360x+x^2} + 5(-360+2x)}{40\sqrt{38025-360x+x^2}}$

$0 = 8\sqrt{38025-360x+x^2} + 5(-360+2x)$

$\left(\frac{1800-10x}{8}\right)^2 = \left(\sqrt{38025-360x+x^2}\right)^2$

$\frac{3240000-36000x+100x^2}{64} = 38025-360x+x^2$

$3240000-36000x+100x^2 = 2433600-23040x+64x^2$

$806400 - 12960x + 36x^2 = 0$

$22400 - 360x + x^2 = 0$

$x = \frac{360 \pm \sqrt{129600-89600}}{2}$

$x = \cancel{280}$ $x = 80$

$T(80) \doteq 47.25s$

Example 7: (Trigonometric)
A right circular cylinder is to be inscribed in a sphere of radius 24 cm. Find the dimensions of the cylinder that will maximize the lateral surface area.

Example 8: (Exponential)
The probability of successfully tossing a beanbag into a basket from x m away is given by $P = (0.85)^x$. The contestant will be paid \$x for every successful shot. Where should they stand to shoot if they want to maximize their revenue?

Example 9: (Cannot Solve Algebraically) -See Lesson 3 computer
What is the area of the largest rectangle that has one corner at the origin and the opposite corner on the curve y = cos(x), where x < pi/2?

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Example 10: (Cannot set up algebraic algebraically) -See Lesson 3 computer
What is the minimum distance between the curves $y = x^2$ & $y = -x^2 + 8x - 12$?

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