Related Rates

When one quantity causes another to change their relationship isn't always linear.

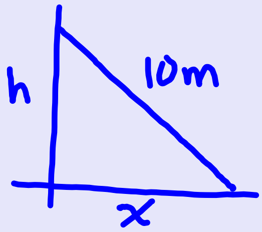
Ex: Double the hours worked → double the pay. (linear, proportional)

But; double the radius of a circle→ 4 times the area. (not linear)

These examples will generally work the same:

- What is our desired (unknown) rate?
- Must have a (at least one) given rate.
- There must be an equation relating the 2(+) quantities.
- Use *Implicit Differentiation* to differentiate the entire equation with respect to time.
- **Lastly:** Substitute in the **specific** values to solve for the unknown rate.

RR Example 1: A ladder 10 m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1m/s, how fast is the top sliding down the wall when the bottom is 6 m from the wall?



$\frac{dh}{dt} = ?$ $\frac{dx}{dt} = 1 m/s$

$h^2 + x^2 = 10^2$

@ $x = 6m$
$h = 8m$

$2h \cdot \boxed{\frac{dh}{dt}} + 2x \cdot \frac{dx}{dt} = 0$

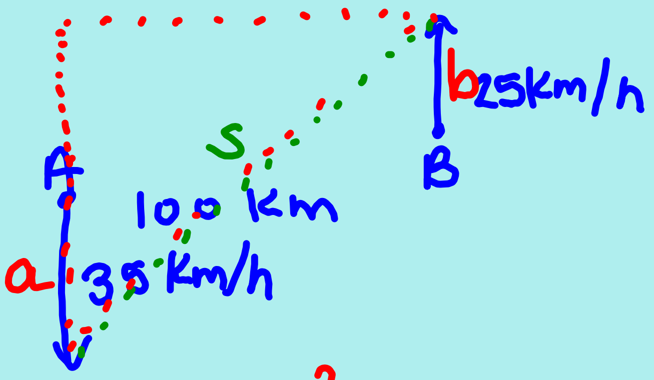
$2(8) \cdot \frac{dh}{dt} + 2(6)(1) = 0$

$16 \cdot \frac{dh}{dt} = -12$

$\frac{dh}{dt} = -\frac{12}{16}$

$\frac{dh}{dt} = -0.75 m/s$

RR Example 2: At noon ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between them changing at 4:00 pm?



$$\frac{da}{dt} = 35\,km/h$$

$$\frac{db}{dt} = 25\,km/h$$

$$\frac{ds}{dt} = ?$$

$$(a+b)^2 + 100^2 = s^2$$

$$a^2 + 2ab + b^2 + 100^2 = s^2$$

$$2a\cdot\frac{da}{dt} + 2(1)\cdot b\cdot\frac{da}{dt} + 2a(1)\cdot\frac{db}{dt} + 2b\cdot\frac{db}{dt} = 2s\cdot\boxed{\frac{ds}{dt}}$$

$$2(140)(35) + 2(100)(35) + 2(140)(25) + 2(100)(25) = 2(260)\cdot\frac{ds}{dt}$$

$$\frac{ds}{dt} \doteq 55.4\,km/h$$

HW: p569#2, 3, 7, 9, 10, 12, 16, 18