UNIT 5: USING THE DERIVATIVE TO SOLVE PROBLEMS.

Lesson 1: Velocity, Acceleration and General Word Problems

Graphic

Ex 1: The position of a particle moving along the ***x*** - axis is given by $s(t) = t^2 - 10t + 16$.

a) Where is the particle at t = 0 s, t = 2s, t = 5 s?

$s(0) =$
$s(2) =$
$s(5) =$

b) Give the velocity of the particle at t = 3s.

$s'(t) = 2t - 10$

$v(3) =$

c) Give the average velocity in the first 3 seconds.

$$\frac{s(3) - s(0)}{3}$$

$=$

d) When is the particle at rest?

$v(t) = 0$

$t = ?$

e) What is the acceleration of the particle?

$v'(t) = 2$

f) What is the initial position of the particle?

$s(0) = 16$ m

Mar 22-7:48 AM

g) When is the particle speeding up?

$a \cdot v > 0$

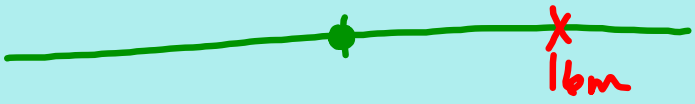
$2 \cdot (2t - 10) > 0$

$2t - 10 > 0$

$t > 5$ s

Mar 22-8:01 AM

h) When is the particle moving toward the origin?



$v(t) = 2t - 10$

$2t - 10 < 0$

$t < 5$

Mar 22-8:01 AM

i) If another particle is released at t = 0 and has a position of $s(t) = 2t^2 - 6t - 5$, what will the velocities of the two particles be when they collide?

$2t^2 - 6t - 5 = t^2 - 10t + 16$

$t^2 + 4t - 21 = 0$

$(t+7)(t-3) = 0$

$t = -7, 3$

$s'(t) = 4t - 6$

$v_2(3) = 4(3) - 6$

$= 6 \text{ m/s}$

$v_1(3) = 2(3) - 10$

$= -4 \text{ m/s}$

Mar 22-8:02 AM

We know from chemistry that at a constant temperature the pressure and volume of a gas are related as follows **PV = k.** Where **k** is a constant determined by the temperature and the mass of the gas. The **isothermal compressibility (β)** of an ideal gas is defined by $\beta = -\frac{1}{V}\frac{dV}{dP}$ For a gas with **k** = 5.3, find the compressibility at P = 40 kPa.

$PV = K$

$PV = 5.3$

$V = \frac{5.3}{P}$

$V(P) = 5.3P^{-1}$

$V(40) = 5.3\left(\frac{1}{40}\right) = \frac{5.3}{40}$

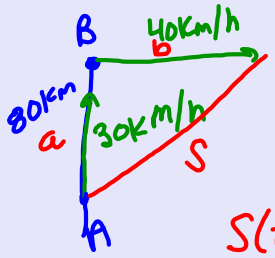
$\frac{dV}{dP} = -5.3P^{-2}$

$\beta = -\frac{1}{V}\cdot\frac{dV}{dP} = \frac{-1}{\frac{5.3}{40}}\cdot\frac{-5.3}{(40)^2}$

$\beta(40) = 0.0251$

Mar 22-8:02 AM

At 1:00 p.m. ship A was 80 km south of ship B. Ship A is sailing north at 30 km/h and ship B is sailing east at 40 km/h. How fast is the distance between them changing at 2:30 p.m.? Are they getting further or closer apart?



$a = 80 - 30t$

$b = 40t$

$s(t) = \sqrt{(80-30t)^2 + (40t)^2}$

$= \sqrt{6400 - 4800t + 2500t^2}$

$s'(t) = \frac{1}{2}(6400 - 4800t + 2500t^2)^{-\frac{1}{2}} \cdot (-4800 + 5000t)$

$s'(1.5) \doteq 19.4 \text{ km/h}$

getting further apart b/c ROC is +ve.

Mar 22-8:04 AM