Curve Sketching

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Algorithm for Sketching any Curve (Page 212)

1. Determine discontinuities and asymptotes
2. Determine intercepts using f(x)
3. Determine critical points using f’(x)
4. Determine intervals of increase/decrease to check if critical points are local maxima, minima, or neither
5. Determine points of inflection and concavity using f’’(x)
6. Determine end behaviours of the function

Keep in Mind

- You won’t need all of these steps in every situation
- You are familiar with the basic shapes of many functions so far. Use this knowledge to your advantage

Example 1

$$f(x) = x^4 - 4x^3 + 4x^2$$

What do you think the graph will look like?

Example 1

$$f(x) = x^4 - 4x^3 + 4x^2$$

1. Discontinuities?
None

2. Intercepts:
y=0, x=0, 2

First derivative: $f'(x) = 4x^3 - 12x^2 + 8x$ $\qquad f'(x) = (4x)(x-2)(x-1)$

3. Critical Points: x = 0, 1, 2
(0,0), (1, 1), (2, 0)

4. Inc/dec

Interval	4x	x-2	x-1	f'(x)	inc/dec
(-inf, 0)	-	-	-	-	dec
(0, 1)	+	-	-	+	inc
(1, 2)	+	-	+	-	dec
(2, inf)	+	+	+	+	inc

So we have mins @ (0, 0), (2, 0) and a max @ (1, 1)

Example 1

$$f(x) = x^4 - 4x^3 + 4x^2$$

Second derivative: $f''(x) = 12x^2 - 24x + 8$ $f''(x) = (x - 0.42)(x - 1.58)$

5. Inflection points: x = 0.42, 1.58

(0.42, 0.44), (1.58, 0.44)

Interval	x-0.42	x-1.58	f''(x)	Concavity
(-inf, 0.42)	-	-	+	up
(0.42, 1.58)	+	-	-	down
(1.58, inf)	+	+	+	up

6. End Behaviours:

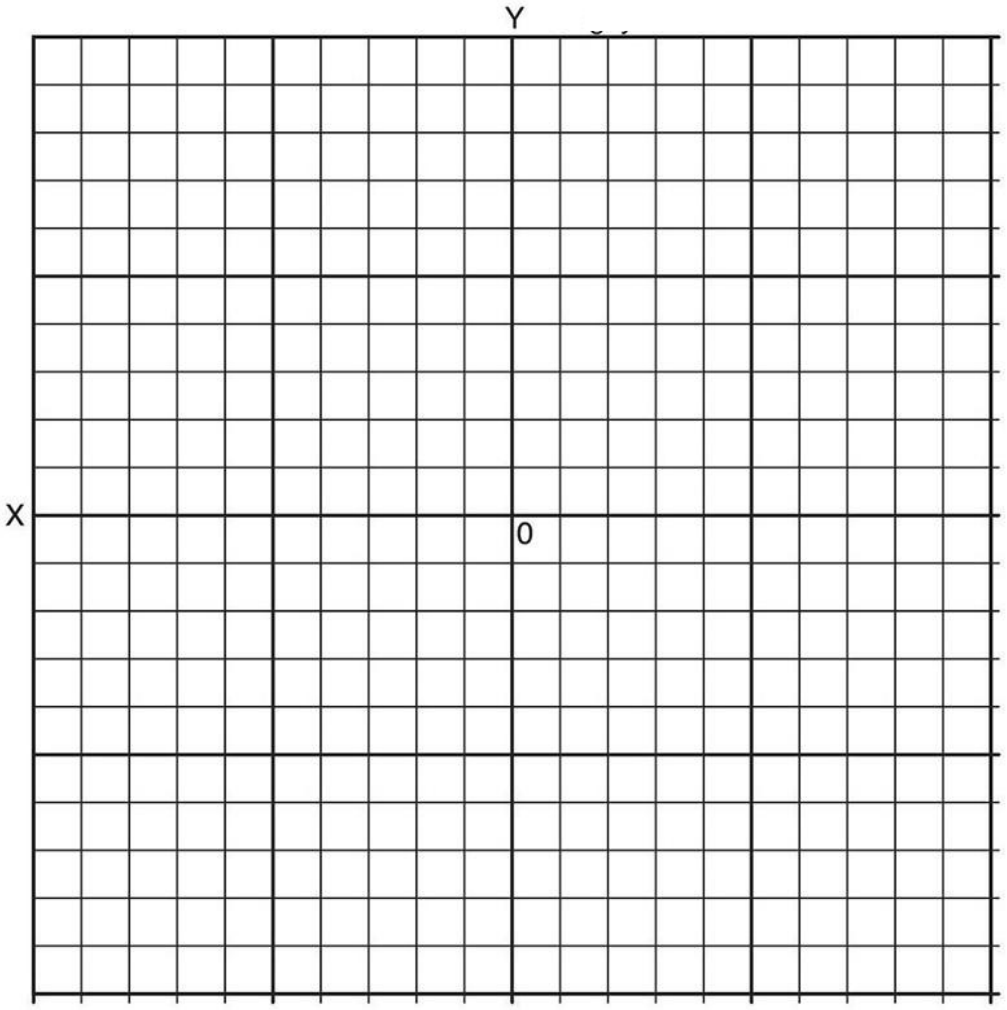
$\{x \to \infty, f(x) \to \infty\}$ $\{x \to -\infty, f(x) \to \infty\}$

Example 1

$$f(x) = x^4 - 4x^3 + 4x^2$$

Final Sketch Steps:

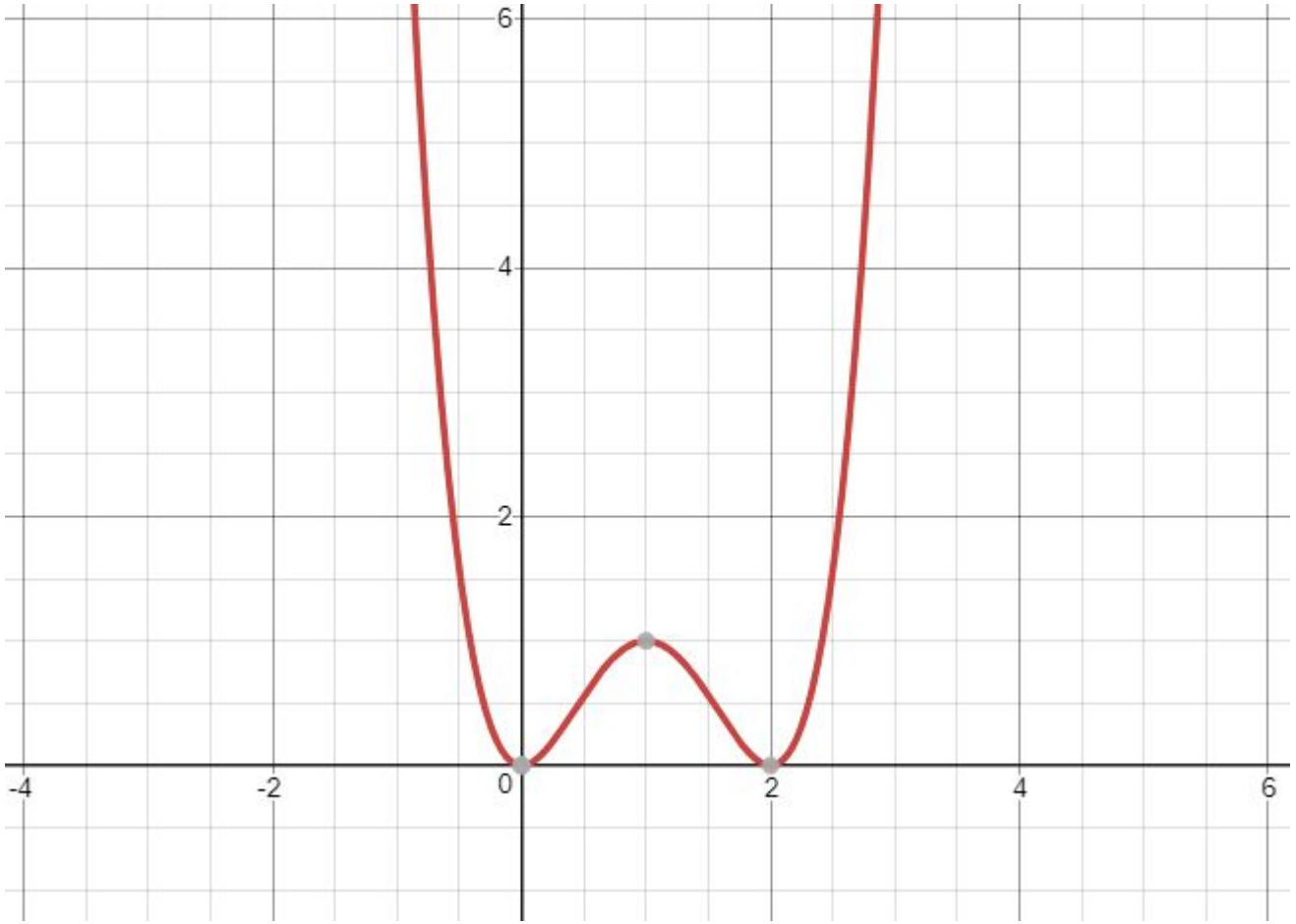
1. Discontinuities
2. X and Y Intercepts
3. Critical Points
4. Inflection Points
5. Connect dots using knowledge of intervals (increasing/ decreasing and concavity) and end behaviour



Example 1

$$f(x) = x^4 - 4x^3 + 4x^2$$

Actual graph:



Example 2

$$f(x) = \frac{x-4}{x^2-4}$$

What do we think it will look like?

Example 2

$$f(x) = \frac{x-4}{x^2-4}$$

1. Asymptotes, Discontinuities:

VAs @ x=2, -2 HA @ y=0

2. Intercepts:

y=1, x=4

First derivative: $f'(x) = -\frac{x^2 - 8x + 4}{(x^2-4)^2}$ $f'(x) = -\frac{(x-7.46)(x-0.54)}{(x^2-4)^2}$

3. Critical Points: x = 0.54, 7.46

(0.54, 0.93), (7.46, 0.07)

4. Inc/dec

Interval	-1	(x-0.54)	(x-7.46)	(x^2-4)^2	f'(x)	inc/dec
(-inf, -2)	-	-	-	+	+	dec
(-2, 0.54)	-	-	-	+	+	dec
(0.54, 2)	-	+	-	+	+	inc
(2, 7.46)	-	+	-	+	-	inc
(7.46, inf)	-	+	+	+	+	dec

min @ (0.54, 0.93) and
max @ (7.46, 0.07)

Example 2

$$f(x) = \frac{x-4}{x^2-4}$$

Second derivative: $f''(x) = \frac{2(-16+12x-12x^2+x^3)}{(-4+x^2)^3}$

5. Inflection points:

Not worth it. Do not gain significant info

6. End behaviours

$\{x \to -\infty, f(x) \to 0\}$ $\{x \to \infty, f(x) \to 0\}$

How is the function approaching 0 at both ends? (Hint: use your calculator)

Ans: -inf from below, +inf from above

Example 2

$$f(x) = \frac{x-4}{x^2-4}$$

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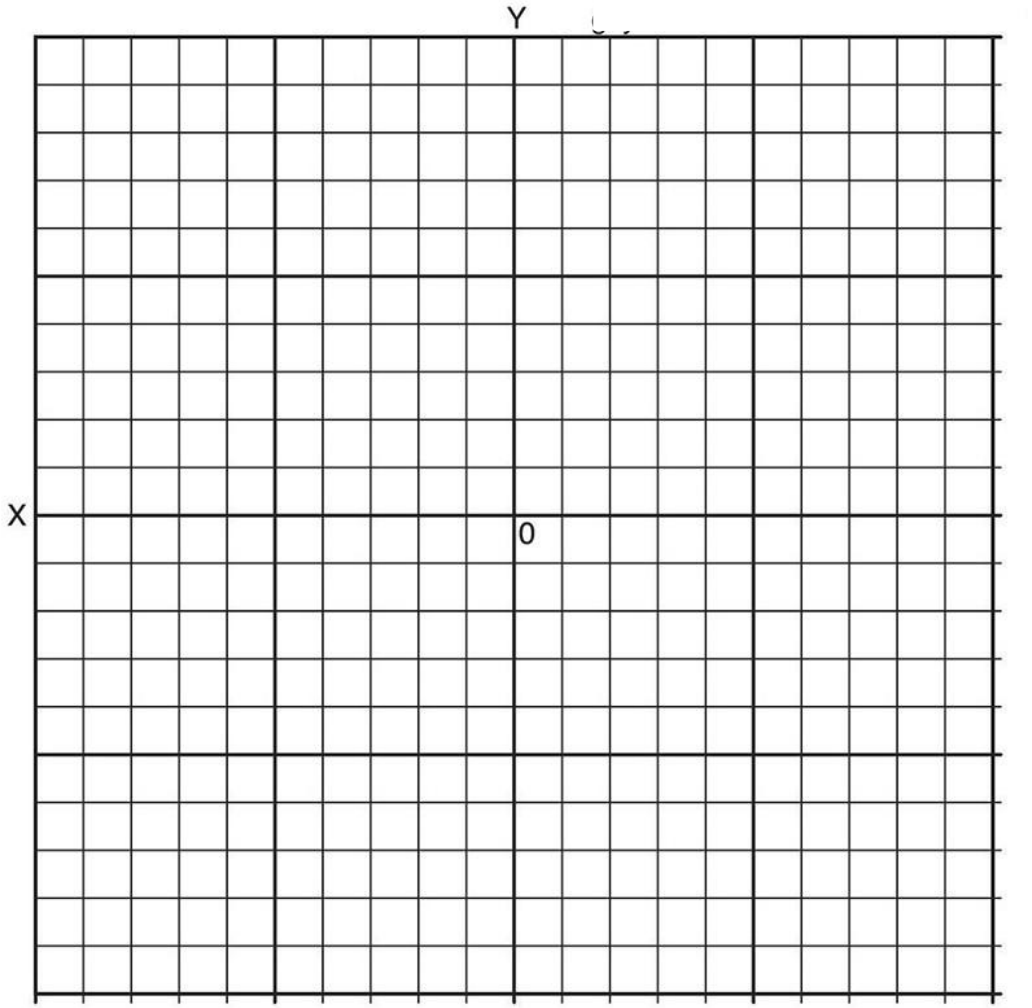

Example 2

$$f(x) = \frac{x-4}{x^2-4}$$

Actual graph: