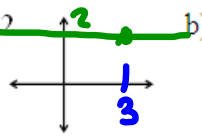
Lesson 2: Asymptotes

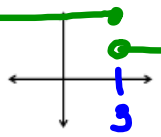
Warm Up: Use your understanding of limits to sketch sections of the given functions around the given points.

a) $\lim_{x\to 3} f(x) = 2$



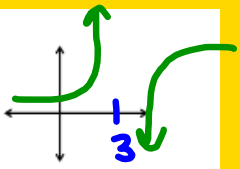
b) $\lim_{x\to 3^-} f(x) = 2$

$\lim_{x\to 3^+} f(x) = 1$



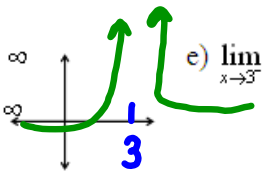
c) $\lim_{x\to 3^-} f(x) = \infty$

$\lim_{x\to 3^+} f(x) = -\infty$

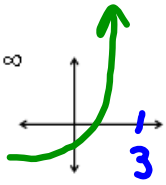


d) $\lim_{x\to 3^-} f(x) = \infty$

$\lim_{x\to 3^+} f(x) = \infty$



e) $\lim_{x\to 3^-} f(x) = \infty$



Vertical Asymptote: A vertical asymptote exists at x = a, if $\lim_{x\to a} f(x) = \pm\infty$

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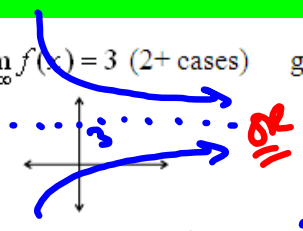
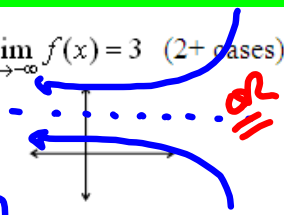
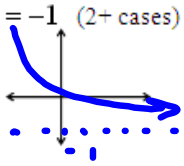
f) $\lim_{x\to\infty} f(x) = 3$ (2+ cases)



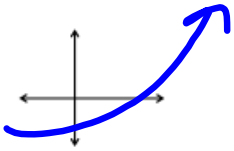
g) $\lim_{x\to-\infty} f(x) = 3$ (2+ cases)



h) $\lim_{x\to\infty} f(x) = -1$ (2+ cases)



i) $\lim_{x\to\infty} f(x) = \infty$



Horizontal Asymptote: A Horizontal Asymptote exists at y = L, if $\lim_{x\to\pm\infty} f(x) = L$

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Strategy: Finding Asymptotes

To find vertical asymptotes: They will occur in places where the limit of the function is infinity. This will only happen at breaks in the curve or at the endpoints.

To find horizontal asymptotes: Check the limit as the function approaches $\pm\infty$.

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Example 1: Evaluate the following limits. Describe the result graphically.

a) $\lim_{x\to 3}\frac{4-\sqrt{x+13}}{x-3}$ $\cdot\ \frac{4+\sqrt{x+13}}{4+\sqrt{x+13}}$

$= \lim_{x\to 3}\frac{16-(x+13)}{(x-3)(4+\sqrt{x+13})}$

$= \lim_{x\to 3}\frac{-(x-3)}{(x-3)(4+\sqrt{x+13})}$

$= -\frac{1}{8}$

hole @ $x=3$

b) $\lim_{x\to 3}\frac{x^2-4x+3}{x^2-6x+9}$

$= \lim_{x\to 3}\frac{(x-3)(x-1)}{(x-3)(x-3)}$

$\frac{2}{0}$

$\therefore$ V.A. @ $x=3$

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Example 1: Evaluate the following limits. Describe the result graphically.

c) $\lim_{x\to\infty} \frac{4-3x-5x^3}{5x-3x^3+4}$

H.A @ $\frac{5}{3}$

d) $\lim_{x\to\infty} \frac{x^2-5x-6}{x-2} = \frac{(x-6)(x+1)}{x-2}$

$$\begin{array}{c|ccc} 2 & 1 & -5 & -6 \\ & & 2 & -6 \\ \hline & 1 & -3 & -12 \end{array}$$

$\therefore$ OA @ $y = x-3$

VA @ $x = 2$

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Example 2: Determine the asymptotes of the following graphs:

a) $y = \frac{x-2}{x^2-2x-3} = \frac{(x-2)}{(x-3)(x+1)}$

horizontal @ $x = 0$

V.A @ $y = 3, y = -1$

OA $\rightarrow$ DNE.

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Example 2: Determine the asymptotes of the following graphs:

b) $y = \dfrac{3x^2+3}{x^2-3x+2}$ $= \dfrac{3(x^2+1)}{(x-2)(x-1)}$

V.A. @ $x=2, x=1$

H.A @ $y=3$

OA DNE.

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Example 2: Determine the asymptotes of the following graphs:

c) $y = \dfrac{2x}{\sqrt{x+3}} \cdot \dfrac{\sqrt{x+3}}{\sqrt{x+3}} = \dfrac{2x(\sqrt{x+3})}{x+3}$

VA @ $x=-3$

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