Lesson 4: Implicit Differentiation & Related Rates

Warm Up: Differentiate the following.

a) $12x - 3y = 18$ b) $x^2 + y^2 = 36$ c) $xy = 3y + 6$ d) $y^3 - y^2 = 4x$

$\frac{-3y}{-3} = \frac{18}{-3} - \frac{12x}{-3}$

$y = -6 + 4x$

$y' = 4$

$y^2 = 36 - x^2$

$y = \pm\sqrt{36 - x^2}$

$y' = \pm\frac{1}{2}(36 - x^2)^{-\frac{1}{2}} \cdot (-2x)$

$xy - 3y = 6$

$y(x-3) = 6$

$y = \frac{6}{x-3}$

$y' =$

IDEA!!: Use implicit differentiation.

Recall: The derivative of x with respect to x is $\frac{dx}{dx} = 1$

The derivative of y with respect to x is dy/dx

Now: Differentiate left to right, accounting for all other rules.

Solve the linear equation for y'.

Example 1: Differentiate the following.

a) $12x - 3y = 18$ b) $x^2 + y^2 = 36$ c) $xy = 3y + 6$ d) $y^3 - y^2 = 4x$

a) $12x-3y=18$

$12 - 3\cdot\frac{dy}{dx} = 0$

$\frac{-3}{-3}\frac{dy}{dx} = \frac{-12}{-3}$

$\frac{dy}{dx} = 4$

b) $x^2+y^2=36$

$2x\cdot\frac{dx}{dx} + 2y\cdot\frac{dy}{dx} = 0$

$\frac{2y}{2y}\cdot\frac{dy}{dx} = \frac{-2x}{2y}$

$\frac{dy}{dx} = \frac{-2x}{2y}$

$\frac{dy}{dx} = \frac{-2x}{\pm\sqrt{36-x^2}}$

c) $xy = 3y+6$

$1\cdot\frac{dx}{dx}\cdot y + x\cdot\frac{dy}{dx} = 3\cdot\frac{dy}{dx}$

$y + x\cdot\frac{dy}{dx} = 3\frac{dy}{dx}$

$y = 3\frac{dy}{dx} - x\cdot\frac{dy}{dx}$

$y = \frac{dy}{dx}(3-x)$

$\frac{y}{3-x} = \frac{dy}{dx}$

$\frac{\frac{6}{x-3}}{3-x} = \frac{dy}{dx}$

$\frac{6}{(x-3)(3-x)} = \frac{dy}{dx}$

d) $y^3 - y^2 = 4x$

$3y^2\cdot\frac{dy}{dx} - 2y\cdot\frac{dy}{dx} = 4$

$\frac{dy}{dx}(3y^2-2y) = 4$

$\frac{dy}{dx} = \frac{4}{3y^2-2y}$

Example 2: (**APPS**) Calculate the equation(s) of the **normal(s)** to $y^2 - 3xy = -5$, at $x = 2$.

$$y^2 - 3xy = -5$$

$y = ?$

$y^2 - 6y + 5 = 0$

$(y-5)(y-1) = 0$

$y = 5 \quad y = 1$

$$2y \cdot \frac{dy}{dx} - 3y - 3x \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2y - 3x) - 3y = 0$$

$$\frac{dy}{dx} = \frac{3y}{2y - 3x} \quad \text{slope of tangent}$$

HW: P564#2ace, 3ac, 5, 7, 10, 12, 15

$$\text{slope of normal} = \frac{-(2y - 3x)}{3y}$$

$$@(2,5) = -\frac{4}{15}$$

$$@(2,1) = \frac{4}{3}$$

eqn 1 @ $(2,5)$, $m = -4/15$

$y = mx + b$

$5 = -\frac{4}{15}(2) + b$

$\frac{83}{15} = b$

$\therefore y = -\frac{4}{15}x + \frac{83}{15}$

eqn 2 @ $(2,1)$, $m = \frac{4}{3}$

$y = mx + b$

$1 = \frac{4}{3}(2) + b$

$-\frac{5}{3} = b$

$\therefore y = \frac{4}{3}x - \frac{5}{3}$

Related Rates

When one quantity causes another to change their relationship isn't always linear.

Ex: Double the hours worked → double the pay. (linear, proportional)

But; double the radius of a circle→ 4 times the area. (not linear)

These examples will generally work the same:

- What is our desired (unknown) rate?
- Must have a (at least one) given rate.
- There must be an equation relating the 2(+) quantities.
- Use *Implicit Differentiation* to differentiate the entire equation with respect to time.
- **Lastly:** Substitute in the **specific** values to solve for the unknown rate.

RR Example 1: A ladder 10 m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1m/s, how fast is the top sliding down the wall when the bottom is 6 m from the wall?

Apr 19-8:31 AM

RR Example 2: At noon ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between them changing at 4:00 pm?

HW: p569#2, 3, 7, 9, 10, 12, 16, 18

Apr 1-12:07 PM