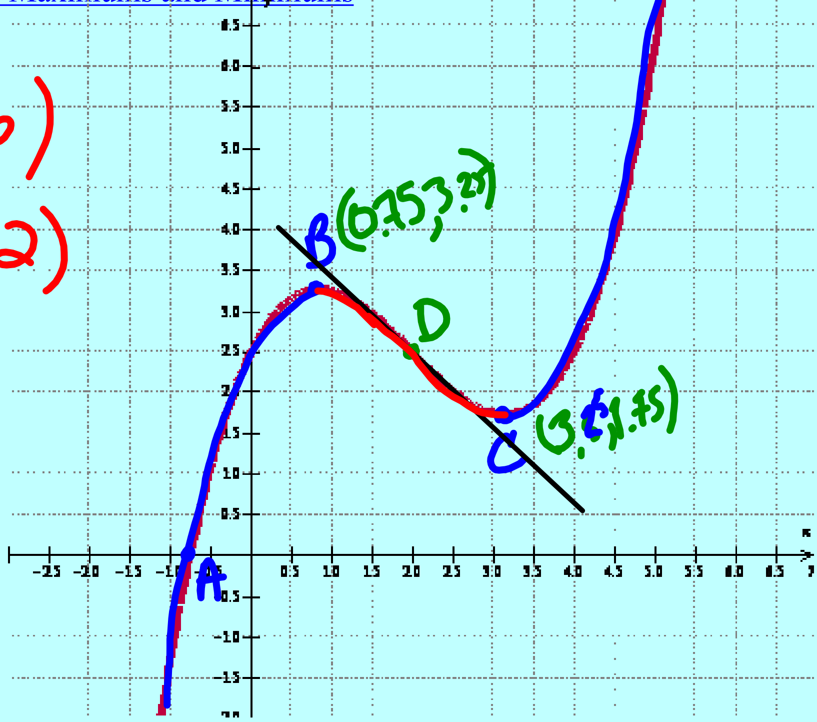
UNIT 5: CURVE SKETCHING

Lesson 1: Increase - Decrease, Maximums and Minimums

C.U. $x \in (2, \infty)$

C.D. $x \in (-\infty, 2)$



Notation:

[illegible]

Increasing: A function, f(x), is increasing on (a, b) if

[illegible]

Decreasing: A function, f(x), is decreasing on (a, b) if

[illegible]

Local Maximum: A local maximum occurs at x = **a** if

[illegible]

Local Minimum: A local minimum occurs at x = **a** if

[illegible]

Concave Up: ‘[illegible]

Concave Down: ‘[illegible]

Point of Inflection: [illegible]

For any function $y = f(x)$,

Increasing/Decreasing:

Increasing: $f'(x) > 0$

Decreasing: $f'(x) < 0$

Ex 1: Determine the intervals of increase and decrease for:

a) $y = x^3 - 12x$

$y' = 3x^2 - 12$

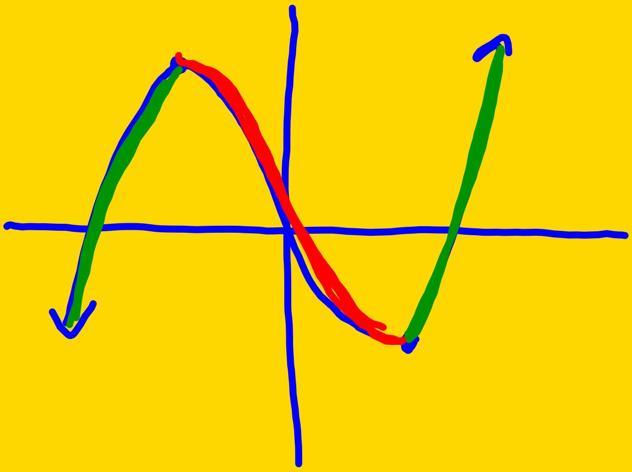
$3x^2 - 12 = 0$

$3(x^2 - 4) = 0$

$x = \pm 2$

$x = 2, \; y = -16$

$x = -2, \; y = 16$



Ex 1: Determine the intervals of increase and decrease for:

b) $y = \dfrac{x}{x^2+1}$

$$y' = \frac{(1)(x^2+1) - (x)(2x)}{(x^2+1)^2}$$

$$0 = \frac{-x^2+1}{(x^2+1)^2} \quad x \in \mathbb{R}$$

$$-x^2+1=0$$
$$x^2-1=0 \rightarrow x = \pm 1$$

Interval	$(1-x^2)$	$(x^2+1)^2$	$f'(x)$	$\therefore f$
$(-\infty,-1)$	$-$	$+$	$-$	dec.
$(-1,1)$	$+$	$+$	$+$	inc.
$(1,+\infty)$	$-$	$+$	$-$	dec.

$\therefore$ f(x) is inc. on $(-1,1)$

f(x) is dec on $(-\infty,-1)$ and $(1,+\infty)$

Ex 1: Determine the intervals of increase and decrease for:

c) $f(x) = x + \sin(x)$

$f'(x) = 1 + \cos x$

$1 + \cos x = 0$

$\cos x = -1$

$x = \pm\pi, \pm 3\pi, \pm 5\pi \ldots\ldots$

Int.	$1+\cos x$	$f'(x)$	$\therefore f(x)$
$(-\pi, \pi)$	+	+	inc
$(\pi, 3\pi)$	+	+	inc
$(3\pi, 5\pi)$	+	+	inc

$\therefore f(x)$ is increasing everywhere

But still has tangents @ $x = \pm\pi, \pm 3\pi, \ldots$

First Derivative Test:

If $f'(a) = 0$ and $f'(x)$ changes sign then we have a local maximum or a local minimum at $x = a$.

If $f'(x)$ changes from + to -, we have a local Max

If $f'(x)$ changes from - to +, we have a local Min

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Example 2: Use the intercepts, extreme values and the first derivative test to sketch $y = x^2 e^x$

$y' = 2xe^x + x^2e^x$

$2xe^x + x^2e^x = 0$

$e^x x(2+x) = 0$

$x = 0,\ x = -2$

Int	e^x	x	$2+x$	$f'(x)$	$\therefore f(x)$
$(-\infty, -2)$	+	−	−	+	inc.
$(-2, 0)$	+	−	+	−	dec.
$(0, +\infty)$	+	+	+	+	inc.

x-int, $y=0$
$0 = x^2e^x$
$x=0$

y-int, $x=0$
$y = 0^2e^0$
$= 0$

$f(0) = 0$ min

$f(2) = 0.54$ max