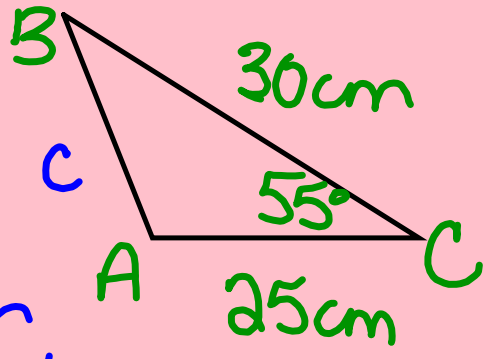
Unit 6: Vectors

Lesson 1: Vectors and Vector Addition

Warm Up: Solve ΔABC if a = 30 cm, b = 25 cm, and <C = 55°

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{30}{\sin A} = \frac{25}{\sin B} = \frac{c}{\sin 55}$



$c^2 = a^2 + b^2 - 2ab\cos C$

$= 30^2 + 25^2 - 2(30)(25)\cos 55°$

$c \doteq 25.8$ cm

$\frac{30}{\sin A} = \frac{25.8}{\sin 55}$

$\angle A \doteq 72.3°$

$\therefore \angle B = 52.5°$

Scalars: A measure with magnitude only (number) (ex: speed)

Vectors: A number with magnitude & direction (measure) (ex: velocity)

Equal Vectors: vectors with equal magnitude in the same direction

Opposite Vectors: vectors with equal magnitude but opp. direction

Parallel Vectors: vector with equal or opp direction

Scalar Multiples: parallel vectors $\vec{u} = 2\vec{v}$

Unit Vectors: a vector with magnitude = 1 $\hat{u}$

Zero Vector: a vector with magnitude = 0 $\vec{0}$

point to point

Notation: Vectors $\vec{v}$, $\mathbf{v}$, $\overrightarrow{AB}$

$|\vec{v}|$, means the magnitude of $\vec{v}$,

$\hat{v}$ is the unit vector in the direction of $\vec{v}$,

$|\vec{v}| = 32\text{km}$

$|\hat{v}| = 1\text{km}$

$$\hat{v} = \frac{1}{|\vec{v}|} \cdot \vec{v}$$

$=$

Example 1: Given a square ABCD, with sides 5 cm. M is the midpoint of BC. E is the intersection of AC and DB

a) Determine the angle between each pair of vectors

i) **AC** and **AD** ii) **AE** and **AD** iii) **AC** and **AE**

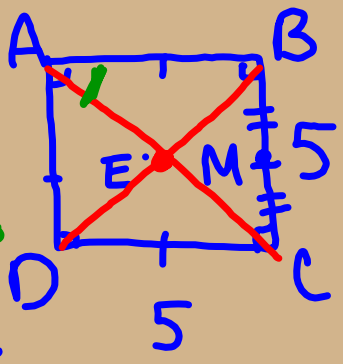
b) Give a vector equivalent to **AB**.

c) Give two vectors parallel to **CD**.

d) Give a vector opposite to **BC**.

e) Find vectors ***u*** and ***v*** such

i) $\boldsymbol{u} = -\boldsymbol{v}$ ii) $\boldsymbol{u} = 0.5\boldsymbol{v}$ iii) $\boldsymbol{u} = -2\boldsymbol{v}$



a) i) 45° ii) 45° iii) 0°

b) $\vec{AB} = \vec{DC} = 2\vec{EM} = -\vec{CD}$

c) $\vec{CD} \parallel \vec{AB} \parallel \vec{EM} \parallel \vec{BA}$

d) $\vec{DA}$

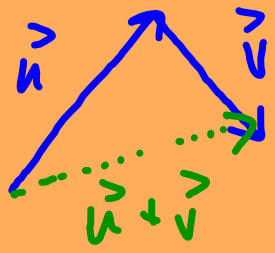
e) i) $\vec{u} = \vec{BC}$ $\vec{v} = \vec{DA}$ ii) $\vec{u} = \vec{EM}$ $\vec{v} = \vec{AB}$ OR $\vec{DC}$

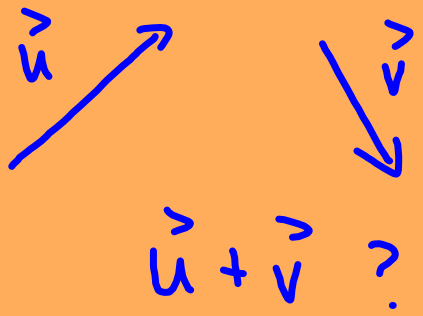
iii) $\vec{v} = \vec{EM}$ $\vec{u} = \vec{BA}$ OR $\vec{CD}$

Triangle Law of Vector Addition

Draw the two vectors head to tail. The sum (resultant) is the vector from the tail of the first to the head of the second. (the short cut!)

Ex:





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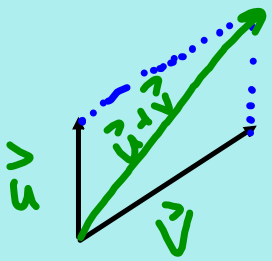
Parallelogram Law of Vector Addition
Draw the two vectors tail to tail. Complete the parallelogram, with these vectors as sides. The resultant is the diagonal.
Ex:



Properties of Vector Addition

a) **Associative** $(\vec{u}+\vec{v})+\vec{w}=\vec{u}+(\vec{v}+\vec{w})$

b) **Commutative** $\vec{u}+\vec{v}=\vec{v}+\vec{u}$

Properties of Scalar Multiplication

a) **Associative** $a\vec{v}=\vec{v}a$

b) **Distributive** $a(\vec{u}+\vec{v})=a\vec{v}+a\vec{u}$

Properties of the Zero Vector

a) Adding the zero vector $\vec{u}+\vec{0}=\vec{0}+\vec{u}$

b) Existence of negative vector $\vec{u}+(-\vec{u})=\vec{0}$

Triangle Inequality

$$|\vec{u}|+|\vec{v}|\geq|\vec{u}+\vec{v}|$$

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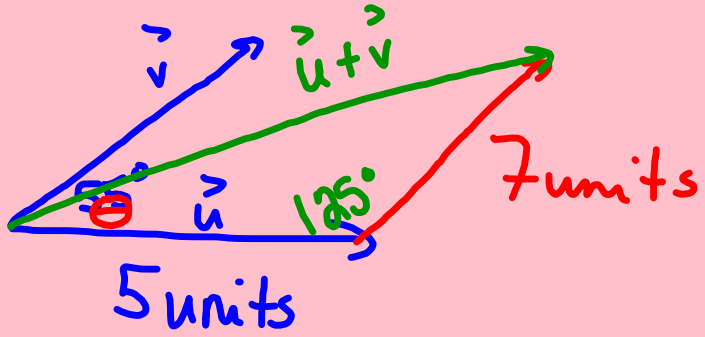
Example 1: Add the vectors ***u*** and ***v***, if $|\vec{u}| = 5, |\vec{v}| = 7$ and the vector **v** lies 55° above vector **u**.



$$|\vec{u}+\vec{v}| = \sqrt{5^2+7^2-2(5)(7)\cos 125}$$
$$\doteq 10.7 \text{ units}$$

$$\frac{\sin\theta}{7} = \frac{\sin 125}{10.7}$$

$$\theta \doteq 32.4°$$

$\therefore \vec{u}+\vec{v}$ is 10.7 units at 32.4° above vector $\vec{u}$.

Example 2: Show that if $|\vec{a}| = |\vec{b}|$ then $\vec{a} + \vec{b}$ is perpendicular to $\vec{a} - \vec{b}$.

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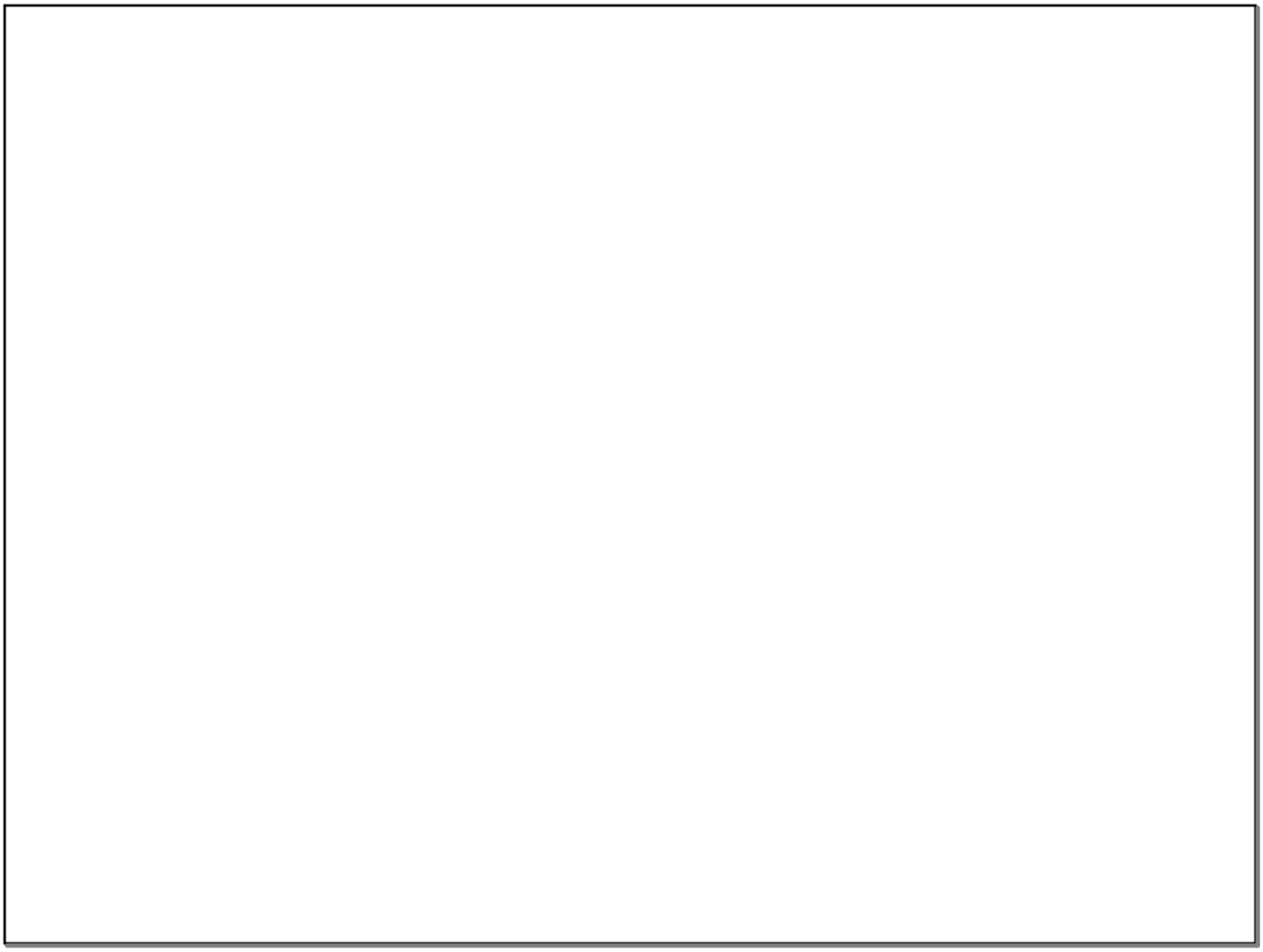

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