PART B: 3-Space Rules

3-D Pythagorean Theorem $|u| = \sqrt{u_x^2 + u_y^2 + u_z^2}$

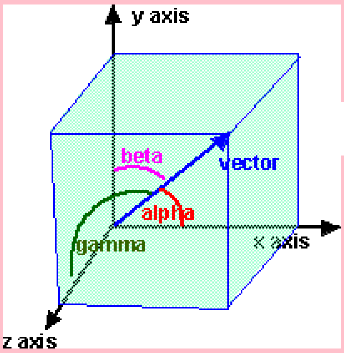
Direction Angles: The set of angles α, β, and γ, a vector<a, b, c> makes with the positive coordinate axes. (x, y and z, respectively) α, β, and γ all lie between zero and 180^{o}

Direction Cosines: The cosines of the direction angles

$$\cos\alpha = \frac{a}{|\vec{u}|}$$

$$\cos\beta = \frac{b}{|\vec{u}|}$$

$$\cos\gamma = \frac{c}{|\vec{u}|}$$



$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$$

and

$a \quad b \quad c$

Ex 6: Determine the direction angles and cosines of <3, -2, 7>.

$$\cos\alpha = \frac{a}{|u|}$$

$$\cos\beta = \frac{b}{|u|}$$

$$\cos\gamma = \frac{c}{|u|}$$

$$|u| = \sqrt{a^2+b^2+c^2}$$
$$= \sqrt{3^2+(-2)^2+(7)^2}$$
$$= \sqrt{9+4+49}$$
$$= \sqrt{62} \text{ units}$$

$$\cos\alpha = \frac{3}{\sqrt{62}}$$
$$\alpha = \cos^{-1}\left(\frac{3}{\sqrt{62}}\right)$$
$$\doteq 67.6^\circ$$

$$\cos\beta = -\frac{2}{\sqrt{62}}$$
$$\beta = \cos^{-1}\left(\frac{-2}{\sqrt{62}}\right)$$
$$\doteq 104.7^\circ$$

$$\cos\gamma = \frac{7}{\sqrt{62}}$$
$$\gamma = \cos^{-1}\left(\frac{7}{\sqrt{62}}\right)$$
$$\doteq 27.2^\circ$$

Ex 7: A vector makes an angle of 30° with the x-axis and 120° with the y-axis, what angle does it make with the z-axis?

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2(30) + \cos^2(120) + \cos^2\gamma = 1$$

$$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{-1}{2}\right)^2 + \cos^2\gamma = 1$$

$$\frac{3}{4} + \frac{1}{4} + \cos^2\gamma = 1$$

$$\cos^2\gamma = 1 - \frac{3}{4} - \frac{1}{4}$$

$$\cos^2\gamma = 0$$

$$\cos\gamma = 0$$

$$\gamma = \cos^{-1}(0)$$

$$= 90°$$

PART C: Problems.

Example 1: Given **u** = <2, -3, 5> and **v** = <4, -1, -3> determine the following vectors.

a) 3**u** b) 2**u** - 3**v** c) **v** + $\hat{i}$

a) $3\langle 2,-3,5\rangle$

$=\langle 6,-9,15\rangle$

b) $2\vec{u}-3\vec{v}$

$=2\langle 2,-3,5\rangle-3\langle 4,-1,-3\rangle$

$=\langle 4,-6,10\rangle+\langle -12,+3,+9\rangle$

$=\langle -8,-3,19\rangle$

c) $\vec{v}+\hat{i}$

$=\langle 4,-1,-3\rangle+\langle 1,0,0\rangle$

$=\langle 5,-1,-3\rangle$

Example 2: Prove that points A(-2, 5), B(4, 3) and C(-5, 6) are collinear.

$\overleftarrow{AB} \parallel \overleftarrow{BC}$

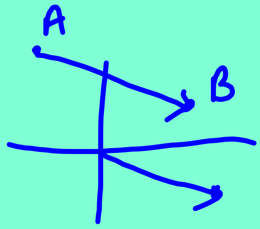
$\overleftarrow{AB} = B - A = (6,-2)$

$\overleftarrow{BC} = C - B = (-9,3)$

$\therefore \vec{AB} = \langle 6,-2 \rangle$

$\vec{BC} = \langle -9,3 \rangle$

$-\frac{9}{6} = -1.5 \quad \frac{3}{-2} = -1.5$



$\therefore -1.5\vec{AB} = \vec{BC}$

$\therefore$ Points, A, B, C are collinear

Example 3: Prove that $|k\vec{v}| = |k| \times |\vec{v}|$

Let $\vec{v} = \langle a, b, c \rangle$

$k\vec{v} = \langle ka, kb, kc \rangle$

$$|k\vec{v}| = \sqrt{(ka)^2 + (kb)^2 + (kc)^2}$$

$$= \sqrt{k^2a^2 + k^2b^2 + k^2c^2}$$

$$= \sqrt{k^2(a^2 + b^2 + c^2}$$

$$= \sqrt{k^2} \cdot \sqrt{a^2 + b^2 + c^2}$$

$$= |k| \cdot |\vec{v}|$$

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