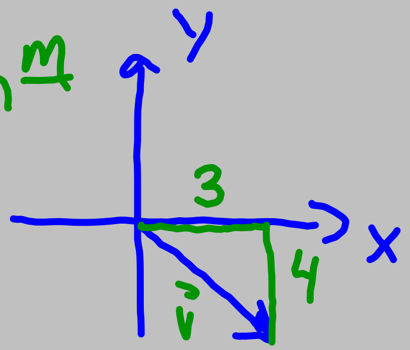
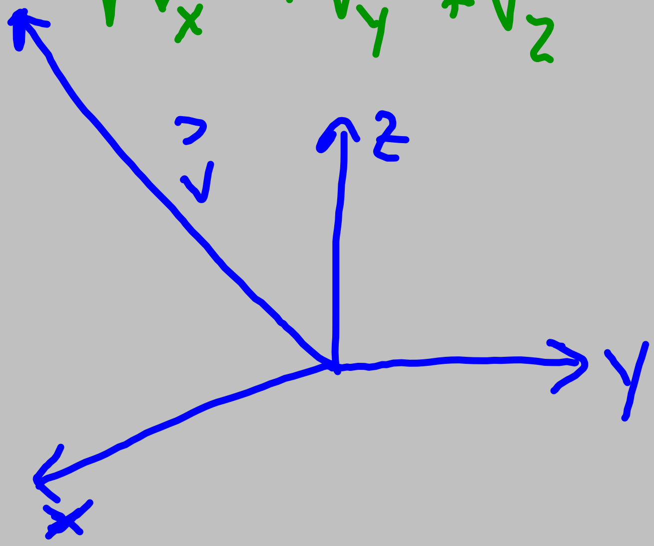
Ex 5: Determine the magnitude of the vector <3, -4, 12>.

3-D Pythagorean Thm



$$|\vec{v}| = \sqrt{\vec{v}_x^{\,2} + \vec{v}_y^{\,2} + V_z^2}$$



$$|\vec{v}| = \sqrt{3^2 + (-4)^2}$$
$$= 5 \text{ units}$$

$$|\vec{V}| = \sqrt{(3)^2 + (-4)^2 + (12)^2}$$
$$= 13 \text{ units}$$

<u>PART B</u>: 3-Space Rules

3-D Pythagorean Theorem $|u| = \sqrt{u_x^2 + u_y^2 + u_z^2}$

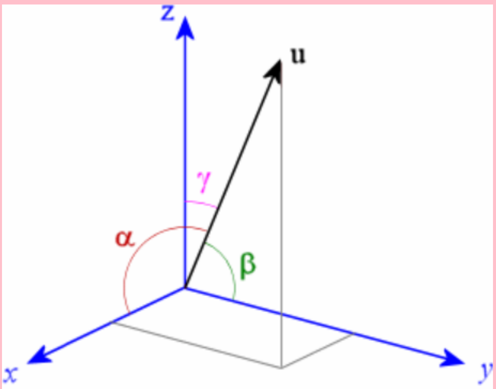
Direction Angles: The set of angles α, β, and γ, a vector <a, b, c> makes with the positive coordinate axes. (x, y and z, respectively) α, β, and γ all lie between zero and 180^{o}

Direction Cosines: The cosines of the direction angles

$\cos\alpha = \frac{a}{|\vec{u}|}$ (with x)

$\cos\beta = \frac{b}{|\vec{u}|}$ (with y)

$\cos\gamma = \frac{c}{|\vec{u}|}$ (with z)



and $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$

Ex 6: Determine the direction angles and cosines of <3, -2, 7>.

$$|\vec{v}| = \sqrt{3^2 + (-2)^2 + (7)^2}$$

$$= \sqrt{62}$$

$$|\vec{v}| \doteq 7.9$$

$$\cos\alpha = \frac{a}{|\vec{v}|} = \frac{3}{7.9} \qquad \cos\beta = \frac{b}{|\vec{v}|} = \frac{-2}{7.9} \qquad \cos\gamma = \frac{c}{|\vec{v}|} = \frac{7}{7.9}$$

$$\alpha \doteq 68^\circ \qquad \beta \doteq 105^\circ \qquad \gamma = 27^\circ$$

Ex 7: A vector makes an angle of 30^{o} with the x-axis and 120^{o} with the y-axis, what angle does it make with the z-axis?

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2 30° + \cos^2 120° + \cos^2\gamma = 1$

$0.75 + 0.25 + \cos^2\gamma = 1$

$\cos^2\gamma = 0$

$\gamma = 90°$

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PART C: Problems.

Example 1: Given **u** = <2, -3, 5> and **v** = <4, -1, -3> determine the following vectors.

a) 3**u** b) 2**u** - 3**v** c) $\mathbf{v} + \hat{i}$

a)
$3\langle 2,-3,5\rangle$
$=\langle 6,-9,15\rangle$

b) $2\langle 2,-3,5\rangle - 3\langle 4,-1,-3\rangle$
$=\langle 4,-6,10\rangle - \langle 12,-3,-9\rangle$
$=\langle -8,-3,19\rangle$

c) $\langle 4,-1,-3\rangle + \langle 1,0,0\rangle$
$=\langle 5,-1,-3\rangle$

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Example 2: Prove that points A(-2, 5), B(4, 3) and C(-5, 6) are collinear.

Method 1.

$m_{\overline{AB}} = -\frac{1}{3}$ $m_{\overline{BC}} = -\frac{1}{3}$

$\therefore m_{\overline{AB}} = m_{\overline{BC}}$

$\therefore$ A, B, C are collinear points.

Method 2

$\vec{AB} = \langle 6, -2 \rangle$ $\vec{BC} = \langle -9, 3 \rangle$

$-1.5\langle 6, -2 \rangle$

$-1.5\vec{AB} = \vec{BC}$

$\therefore$ collinear

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Example 3: Prove that $|k\vec{v}| = |k| \times |\vec{v}|$

$\vec{v} = \langle a, b, c \rangle$

$k\vec{v} = \langle ka, kb, kc \rangle$

$|k\vec{v}| = \sqrt{a^2k^2 + b^2k^2 + c^2k^2}$

$= \sqrt{k^2(a^2 + b^2 + c^2)}$

$= \sqrt{k^2} \cdot \sqrt{a^2 + b^2 + c^2}$

$= |k| \cdot |\vec{v}|$

QED.

May 6-11:19 AM