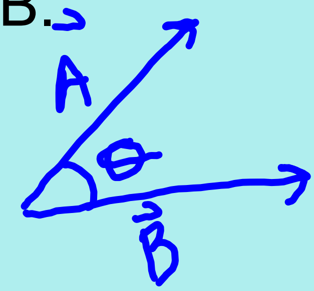
Lesson 7.3 - The Dot (Scalar) Product

The dot product of vectors $\vec{A}$ and $\vec{B}$ is the scalar quantity $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta$ where θ is the angle between the vectors $\vec{A}$ and $\vec{B}$.



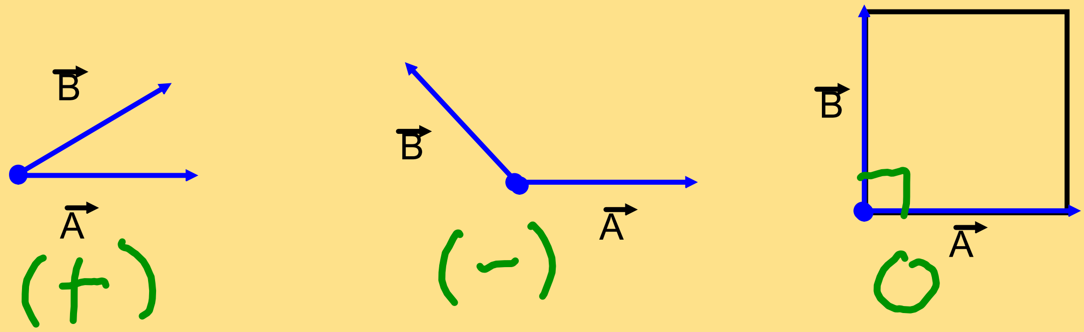
Sign of the Dot Product

The dot product can be positive, negative or zero depending on the size of the angle between the two vectors.

for $0<\theta<90°$, $\cos\theta>0$ so $\vec{A}\cdot\vec{B}>0$

for $\theta=90°$, $\cos\theta=0$ so $\vec{A}\cdot\vec{B}=0$

for $90°<\theta<180°$, $\cos\theta<0$ so $\vec{A}\cdot\vec{B}<0$

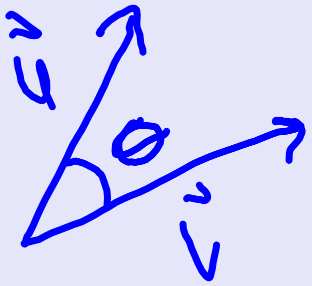


Example 1: Determine the dot product of **u** and **v** where |**u**| = 30 N, and |**v**| = 50 N, and the force are applied at 30º to one another.

$\vec{u} \cdot \vec{v} = |u||v|\cos\theta$

$\vec{u} \cdot \vec{v} = 30_N \cdot 50_N \cdot \cos 30°$

$= 1299$



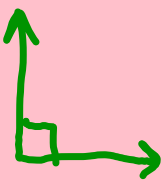
Example 2: Show that for non zero vectors, **u** and **v**, **u•v = 0** if and only if **u** and **v** are perpendicular.
Two Proofs.

Assume **u•v** =0

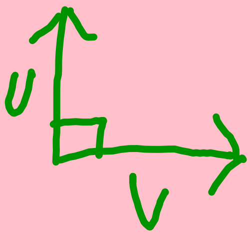
$\vec{u}\cdot\vec{v} = |u||v|\cos\theta = 0$

$|\vec{u}| \neq 0 \quad \cos\theta = 0$

$|\vec{v}| \neq 0 \quad \theta = 90^\circ$



Assume **u** is perpendicular to **v**



$\cos 90 = 0$

$|u||v|\cdot\cos 90 = 0$

Example 3: Compute the dot product of the unit vector **i** with itself, the unit vector **i** with the unit vector **j** and the unit vector **i** with the unit vector **k**.

$\vec{i}\cdot\vec{i} = ||1||\,||1||\cos 0$
$= 1$

$\vec{i}\cdot\vec{j} = ||1||\,||1||\cos 90$
$= 0$

$\vec{i}\cdot\vec{k} = ||1||\,||1||\cos 90$
$= 0$

What do you expect the following to equal?

a) **j•j**= 1

b) **k•k**= 1

c) **j•k**= 0

d) **j•i**= 0

e) **k•i**= 0

f) **k•j**= 0

Dot product of algebraic vectors: $\mathbf{u \cdot v} = u_x v_x + u_y v_y + u_z v_z$

Example 4:

a) Evaluate the dot product of **u** = <1, 2, -3> and **v** = <4, 0, 7>

b) Determine the angle between **u** and **v**.

a) $u \cdot v = 1(4) + 2(0) + (-3)(7)$

$= -17$

$|u| = \sqrt{1+4+9}$

$= \sqrt{14}$

$|v| = \sqrt{65}$

$\underline{u \cdot v = |\vec{u}||\vec{v}|\cos\theta}$

$\theta \doteq 124°$

What is the general formula for the angle between two vectors?

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$$

Example 5: Determine a vector, in the x-z plane that is perpendicular to the vector **u** = <3, -1, -2>.

Hint: What needs to be true for the vectors to be perpendicular? What is the algebraic form of a vector in the x-z plane?

$\vec{u} \cdot \vec{v} = 0$

V be a vector in the x-z plane

$V = \langle a, 0, c \rangle$

$0 = (3 \cdot a) + 0 + (-2 \cdot c)$

$0 = 3a - 2c$

$3a = 2c$

$\frac{3a}{2} = c$ $\quad V = \langle 4, 0, 6 \rangle$

$4 = a$

$6 = c$

$V = \langle 2, 0, 3 \rangle$

Work: The work done by a force **F,** carried through a distance **d** is given by W = **F•d.**

What is the unit for work?

$$W = F \cdot d$$
$$= N \cdot m$$
$$= \text{Joules } (J)$$

Example 6: Find the work done **against gravity** (acting down, along **j**) to move a 10 kg weight from the point (2, 3) to (5, 7).

$d = \langle x, y \rangle$ $\langle 3^m, 4^m \rangle$

$\vec{x} = 5 - 2$
$= 3$

$\vec{y} = 7 - 3$
$= 4$

$F \cdot d = (3 \cdot 0) + (4 \cdot 98)$

$W = 392\,J$

(+)

$F = mg$
$= 98N$

$F = \langle 0, 98N \rangle$

Properties of the Dot Product

a) Commutative: $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}.$

b) Distributive over Vector addition:

$$\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}.$$

c) $(c_1\mathbf{a}) \cdot (c_2\mathbf{b}) = (c_1c_2)(\mathbf{a} \cdot \mathbf{b})$

d) Not Associative (i.e $\mathbf{a} \bullet \mathbf{b} = \mathbf{a} \bullet \mathbf{c}$ does not imply $\mathbf{b} = \mathbf{c}$)

e) Two non-zero vectors **a** and **b** are perpendicular if and only if $\mathbf{a} \bullet \mathbf{b} = 0$

f) $\mathbf{a} \bullet \mathbf{a} = |\mathbf{a}|^2$ (ie. any vector dot itself is equal its magnitude squared)

$\cos\theta = 1$ $|a||a| = |a|^2$

Homework:

Page 377, #1,2,4(oral),6-7ace,9,11,15

Page 387, #6-7a,9a,10,12,16