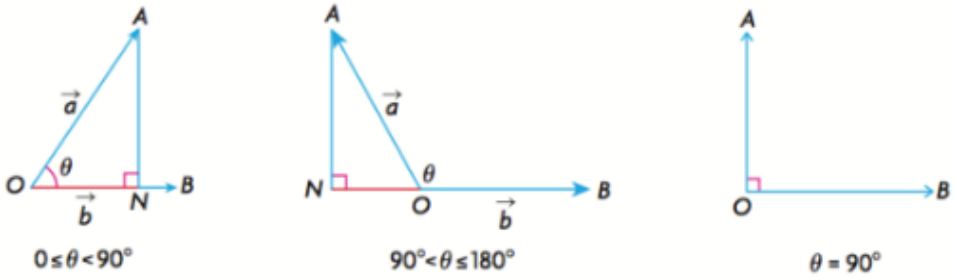
7.5- Scalar & Vector Projections

If **a** and **b** are two vectors, the projection of **a** onto **b** is a vector whose magnitude is equal to $|\mathbf{a}|\cos(\theta)$ (called the component of **a** onto **b**) and is parallel to **b**. It is often thought of as the *shadow* produced by a distant light source parallel to **b**.

$|b|\cos\theta$

Scalar Projection of $\vec{a}$ on $\vec{b}$

The scalar projection of vector $\vec{a}$ onto $\vec{b}$ is ON, where $ON = |\vec{a}|\cos\theta$.



May 8-10:20 AM

The **component** of **b** onto **a** is the scalar quantity given by $\frac{\vec{b} \bullet \vec{a}}{|\vec{a}|}$ = |b|cosθ

Scalar projection

scaler

scaler

What is the component of **a** onto **b?**

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{\vec{b} \cdot \vec{a}}{|\vec{b}|}$$

The scalar component can be converted to a vector along **a** by multiplying the component by $\frac{\vec{a}}{|\vec{a}|}$ where $\frac{\vec{a}}{|\vec{a}|}$ is a unit vector pointing in the direction of **a**

The **projection** of **b** onto **a** is the vector quantity given by $\frac{\vec{b} \bullet \vec{a}}{|\vec{a}|^2}\vec{a}$, denoted by $\text{proj}_{\mathbf{a}}\mathbf{b}$

scaler

vector

scaler

If |**a**|=10, |**b**|=16 and the angle between the two vectors is θ=120 degrees, what is the scalar projection of **a** onto **b**?

$\text{comp}_{\vec{b}}\vec{a} = |10|\cos 120°$

$= -5$

What is the scalar projection of **b** onto **a**?

$\text{comp}_{\vec{a}}\vec{b} = |16|\cos 120$

$= -8$

Give the vector projection of **u**=<2,-3,5> onto **v**=<4,-1,-3>.

$$\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\vec{a} \qquad \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\cdot\vec{v}$$

$\vec{u}\cdot\vec{v} = -4$ $\quad \frac{-4}{\sqrt{26}^2} = \frac{-4}{26}$ $\quad \frac{-4}{26}\langle 4,-1,-3\rangle$

$|\vec{v}| = \sqrt{26}$ $\quad = \left\langle -\frac{8}{13}, \frac{2}{13}, \frac{6}{13}\right\rangle$

$\vec{u}\cdot\vec{v} = u_x v_x + u_y v_y + u_z v_z$

$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$

May 9-6:48 AM

Find the vector projection of $\overrightarrow{OA}=(4,3)$ on $\overrightarrow{OB}=(4,-1)$.

$$\text{Proj}_{\vec{OB}}\vec{OA} = \frac{\vec{OA}\cdot\vec{OB}}{|\vec{OB}|^2}\vec{OB}$$

$$= \frac{(4,3)(4,-1)}{4^2+1}(4,-1)$$

$$= \frac{13}{17}\cdot(4,-1)$$

$$= \left(\frac{52}{17}, \frac{-13}{17}\right)$$

May 9-6:52 AM

Homework: Page 399 #5, 7, 11, 12, 15

May 9-6:55 AM