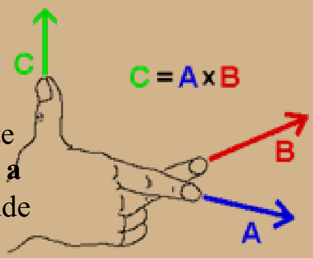
Lesson 8: Cross (Vector) Product of Two Vectors

The Cross Product of vectors **u** and **v** is the vector quantity **u**×**v**

$\left(|\vec{u}||\vec{v}|\sin\theta\right)\hat{n}$ where θ is the angle between the vectors and $\hat{n}$ is the vector normal to both **u** and **v** whose direction is determined by the right hand rule.

Do not Copy (Available Wikipedia, May 2010): In a three-dimensional Euclidean space, with a usual right-handed coordinate system, **a** × **b** is defined as a vector **c** that is perpendicular to both **a** and **b**, with a direction given by the right-hand rule and a magnitude equal to the area of the parallelogram that the vectors span



Adjust A or B (or the plane) and see the cross product change.

Apr 28-10:18 PM

Example 1: Determine the cross product of each pair of unit coordinate vectors.

$\hat{i} \times \hat{i} = |\hat{i}||\hat{i}|\sin\theta \cdot \hat{n}$
$= \vec{0}$

$\hat{i} \times \hat{j} = |\hat{i}||\hat{j}|\sin 90^\circ \hat{n}$
$= \hat{k}$

$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{j} \times \hat{j} = 0$

$\hat{k} \times \hat{k} = 0$

$\hat{k} \times \hat{j} = -\hat{i}$

Apr 28-10:23 PM

Example 2: <u>Explain</u> why **u**×**v** = - **v**×**u**

* Right hand rule.

Apr 28-10:23 PM

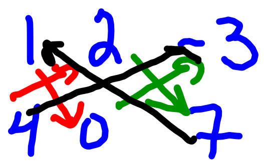
For Algebraic Vectors

$\vec{x} = < x_1, x_2, x_3 >$

$\vec{y} = < y_1, y_2, y_3 >$

$\vec{x} \times \vec{y} = (x_2y_3 - x_3y_2)\hat{i} - (x_1y_3 - x_3y_1)\hat{j} + (x_1y_2 - x_2y_1)\hat{k}$

Example 3: Given **u** = <1, 2, -3> and **v** = <4, 0, 7>, determine **u**×**v.**



$\vec{u} \times \vec{v} = \langle 14, -19, -8 \rangle$

Apr 28-10:29 PM

Example 4: Use a counter example to show that the cross product is not commutative.

$\vec{u} = \langle 3, 1, 4 \rangle$

$\vec{v} = \langle -2, 0, 1 \rangle$

$\vec{u} \times \vec{v}$

$= \langle 1, -11, 2 \rangle$

$$\begin{matrix} 3 & 1 & 4 \\ -2 & 0 & 1 \end{matrix}$$

$\vec{v} \times \vec{u}$

$= \langle -1, 11, -2 \rangle$

$$\begin{matrix} -2 & 0 & 1 \\ 3 & 1 & 4 \end{matrix}$$

Apr 28-10:30 PM

Example 5: Use the vectors $\vec{\mathbf{u}}$ = <a, b, c> and **v** = <r, s, t> to calculate **u**·(**u**×**v**) and **v**·(**u**×**v**). What property of the cross product does this verify?

$\vec{u} \times \vec{v}$

$$\begin{matrix} a & b & c \\ r & s & t \end{matrix}$$

$\vec{u} \cdot (\vec{u} \times \vec{v})$

$\langle a, b, c \rangle \cdot \langle bt - cs, cr - ta, as - rb \rangle$

$= \cancel{abt} - \cancel{acs} + \cancel{bcr} - \cancel{bta} + \cancel{cas} - \cancel{crb}$

$= 0$

$\vec{v} \cdot (\vec{u} \times \vec{v}) = 0$

Apr 28-10:31 PM

Properties of the Cross Product

a) Anticommutative: $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$

b) Distributive over Vector addition: $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) + (\mathbf{a} \times \mathbf{c})$

c) $(r\,\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (r\,\mathbf{b}) = r\,(\mathbf{a} \times \mathbf{b})$

d) Not Associative (i.e. $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$ does not imply $\mathbf{b} = \mathbf{c}$)

e) $\mathbf{a}$ and $\mathbf{b}$ are parallel iff $\mathbf{a} \times \mathbf{b} = \mathbf{0}$

Apr 28-10:32 PM

TRICK

$\vec{u} = \langle 1, 2, -3 \rangle$

$\vec{v} = \langle 4, 0, 7 \rangle$

$$\begin{matrix} \cancel{1} & 2 & -3 & 1 & 2 & \cancel{-3} \\ \cancel{4} & 0 & 7 & 4 & 0 & \cancel{7} \end{matrix}$$

$\langle 14, -19, -8 \rangle$

May 9-10:04 AM