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$\vec{r} = (1, 2, -1)$

$\vec{s} = (-2, -4, 2)$

$-2\vec{r} = \vec{s}$

$0 = (1, 2, -1) \cdot (x, y, z)$

$x + 2y - z = 0$

$x = 3$

$3 + 2y - z = 0$

$z = 2$

$3 + 2y - 2 = 0$

$2y = -1$

$y = -0.5$

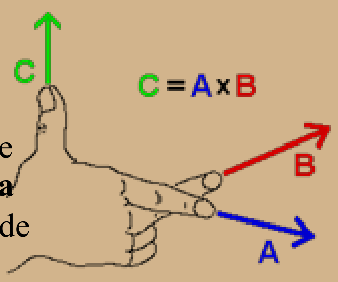
one $\perp$ $\langle 3, -\frac{1}{2}, 2 \rangle$

Lesson 8: Cross (Vector) Product of Two Vectors

The Cross Product of vectors **u** and **v** is the vector quantity **u** × **v** =

$\left(|\vec{u}||\vec{v}|\sin\theta\right)\hat{n}$ where θ is the angle between the vectors and $\hat{n}$ is the vector normal to both **u** and **v** whose direction is determined by the right hand rule.

Do not Copy (Available Wikipedia, May 2010): In a three-dimensional Euclidean space, with a usual right-handed coordinate system, **a** × **b** is defined as a vector **c** that is perpendicular to both **a** and **b**, with a direction given by the right-hand rule and a magnitude equal to the area of the parallelogram that the vectors span



Adjust A or B (or the plane) and see the cross product change.

Example 1: Determine the cross product of each pair of unit coordinate vectors.

$\hat{i} = \langle 1, 0, 0 \rangle$

$\hat{j} = \langle 0, 1, 0 \rangle$

$\hat{k} = \langle 0, 0, 1 \rangle$

$\hat{i} \times \hat{i} = |\hat{i}||\hat{i}| \sin 0 \, \hat{n}$

$= 0$

$\hat{j} \times \hat{j} = 0$

$\hat{k} \times \hat{k} = 0$

$\hat{i} \times \hat{k} = (1)(1) \sin 90 \, \hat{n}$

$= 1 \, \hat{n}$

$= -\hat{j}$

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{j} = -\hat{i}$

Example 2: Explain why $\mathbf{u} \times \mathbf{v} = -\mathbf{v} \times \mathbf{u}$

* Right-hand rule gets reversed.

For Algebraic Vectors

$\vec{x} = < x_1, x_2, x_3 >$

$\vec{y} = < y_1, y_2, y_3 >$

$\vec{x} \times \vec{y} = (x_2 y_3 - x_3 y_2)\hat{i} - (x_1 y_3 - x_3 y_1)\hat{j} + (x_1 y_2 - x_2 y_1)\hat{k}$

Example 3: Given **u** = <1, 2, -3> and **v** = <4, 0, 7>, determine **u** × **v**.

Method 1

$$\begin{matrix} 1 & 2 & -3 \\ 4 & 0 & 7 \end{matrix}$$

$\vec{u} \times \vec{v} = \langle 14, -19, -8 \rangle$

Method 2

$$\begin{matrix} \cancel{1} & 2 & -3 & 1 & 2 & \cancel{-3} \\ \cancel{4} & 0 & 7 & 4 & 0 & \cancel{7} \end{matrix}$$

$\langle 14, -19, -8 \rangle$

Ex: $\vec{a} = \langle 2, 3, -4 \rangle$

$\vec{b} = \langle 1, -8, 2 \rangle$

$$\begin{array}{cccccc} \cancel{2} & 3 & -4 & 2 & 3 & \cancel{-4} \\ \cancel{1} & -8 & 2 & 1 & -8 & \cancel{2} \end{array}$$

$\vec{a} \times \vec{b} = \langle -26, -8, -19 \rangle$

$\vec{b} \times \vec{a}$

$$\begin{array}{cccccc} \cancel{1} & -8 & 2 & 1 & -8 & \cancel{2} \\ \cancel{2} & 3 & -4 & 2 & 3 & \cancel{-4} \end{array}$$

$= \langle 26, 8, 19 \rangle$

Example 4: Use a counter example to show that the cross product is not commutative.

in the last example

we just proved that

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

Example 5: Use the vectors **u**= <a, b, c> and **v** = <r, s, t> to calculate **u**(**u**×**v**) and **v**(**u**×**v**). What property of the cross product does this verify?

$$\vec{u} \times \vec{v} = \langle bt - sc, cr - at, as - rb \rangle$$

$$\begin{matrix} \cancel{a} & b & c & a & b & \cancel{c} \\ \cancel{r} & s & t & r & s & \cancel{t} \end{matrix}$$

$$\vec{u} \cdot (\vec{u} \times \vec{v}) = \langle a, b, c \rangle \cdot \langle bt - sc, cr - at, as - rb \rangle$$

$$= \cancel{abt} - \cancel{asc} + \cancel{bcr} - \cancel{bat} + \cancel{cas} - \cancel{crb}$$

$$= 0$$

$$\vec{v} \cdot (\vec{u} \times \vec{v}) = 0$$

Properties of the Cross Product

a) Anticommutative: $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$

b) Distributive over Vector addition: $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) + (\mathbf{a} \times \mathbf{c})$

c) $(r\,\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (r\,\mathbf{b}) = r\,(\mathbf{a} \times \mathbf{b})$

d) Not Associative (i.e. $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$ does not imply $\mathbf{b} = \mathbf{c}$)

e) $\mathbf{a}$ and $\mathbf{b}$ are parallel iff $\mathbf{a} \times \mathbf{b} = \mathbf{0}$