Lesson 1: Lines in a Plane (Parametric, Vector and Scalar)

Warm Up: Consider the line $y = \frac{2}{3}x + 5$,

A) What does the equation tell us?

B) What does the slope tell us, practically?

C) What is the easiest way to graph the line?

D) What if it was described as an instruction to follow it? How could that be done?

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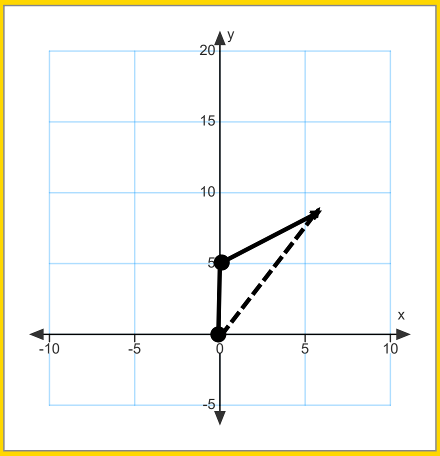

A) The equation tells us that we start at 5, and go up 2 and over 3 units to get subsequent points.

B) Slope practically tells us the direction to move

C) Use the y-intercept as starting place and the slope to find subsequent points

D) Start at (0, 5) then go over 3, up 2, and repeat

New Interpretation: This line is the set of points that:
Start at (0, 5) walk in the ratio 3 over and 2 up
i.e. Go to the point (0, 5), then stay parallel to the vector $\langle 3, 2\rangle$
Or any point P(x, y) on the line has position vector

$$\langle x, y\rangle = \langle 0, 5\rangle + t\langle 3, 2\rangle,\ t \in \Re$$

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This is the vector form of the line

$$\vec{r} = \langle x_0, y_0 \rangle + t\langle a, b \rangle, \quad t \in \Re$$

$\vec{r} = \langle x, y \rangle$ is the position vector of any point on the line

$\langle x_0, y_0 \rangle$ is the position vector of a particular point on the line

$\langle a, b \rangle$ is the direction vector of the line.

This leads to the Parametric Form:

$$x = x_0 + ta$$
$$y = y_0 + tb$$

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Ex 0: Give three points on each line

a) $\langle x, y \rangle = \langle -3, 2 \rangle + t\langle 2, -1 \rangle$ (*i.e. pick 3 values for t*)

b) $x = 4 - 2t$

$y = 6 + t$

a) $t=1$, $(-1, 1)$

$t=2$, $(1, 0)$

$t=3$, $(3, -1)$

b) @ $t=1$, $(2, 7)$

$t=2$, $(0, 8)$

$t=3$, $(-2, 9)$

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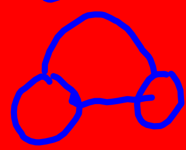
Ex 1: Complete the table

Scalar	Vector	Parametric
y = 2x - 4	$\vec{r}=\langle 0,-4\rangle + t\langle 1,2\rangle$	$x = t$ $y = -4+2t$
$y=\frac{7}{5}x+10.85$	$\vec{r}=\langle 8,-4\rangle + t\langle 5,7\rangle$	$x = 8+5t$ $y = -4+7t$
$y=\frac{-2}{-6}x-1$	$\vec{r}=\langle 3,0\rangle + t\langle -6,-2\rangle$	x = 3 - 6t y = -2t

$y=\frac{1}{3}x+b$
$0=\frac{1}{3}(3)+b$
$-1=b$

$y=mx+b$
$-4=\frac{7}{5}(8)+b$
$-\frac{76}{5}=b$



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Example 2: Are these two lines coincident? (i.e. the same?)

L_1
x = 1 - 2t
y = -2 + t

L_2 $\vec{r}=\langle 1,1\rangle + t\langle 8,-4\rangle$

$\vec{d}_1=\langle -2,1\rangle$ $\vec{d}_2=\langle 8,-4\rangle$

$-4\vec{d}_1=\vec{d}_2$ ∴ they are parallel

Check if (1,1) lies on L_1

$1=1-2t$ $1=-2+t$
$0=t$ $3=t$

∴ Lines are parallel
But <u>NOT</u> coincident.

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Ex 3: Determine a normal to the line Ax + By + C = 0.

* normal lines have -ve reciprocal slopes.

Find $\vec{d}$ {direction vector/slope}

$Ax + By + C = 0$

$\frac{\cancel{B}y}{\cancel{B}} = -\frac{A}{B}x - \frac{C}{B}$

$\vec{d} = \langle B, -A \rangle$

Let $\vec{N} = \langle n_x, n_y \rangle$

$\vec{N} \cdot \vec{d} = 0$

$\langle n_x, n_y \rangle \cdot \langle B, -A \rangle = 0$

$Bn_x - An_y = 0$

Let $n_x = A$, $\therefore n_y = B$

$\therefore \vec{N} = \langle A, B \rangle$

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