Lesson 3: Equation of Plane in 3-Space (Vector and Parametric) Link to plotter

Complete the table:

Equation	Form	Point (x, y, z)	Dimension (What?)
$2x - 4y + 8 = 0$	Standard	(2, 3)	R^2 line
$\vec{r} = <-1,3,7> + t<1,4,-2>$ $t=3$	vec	(2, 15, 1)	R^3 line
$x = -2t$ $y = 3 - t$ $z = 8 + 3t$	para	(4, 5, 2)	R^3 line
$\frac{x-3}{7} = 2y = z - 6$ $t = -2$	symm	(-11, -1, 4)	R^3 line
$\vec{r} = <1,-2,0> + s\underbrace{<2,0,1>} + t\underbrace{<-1,1,1>}$	vect	(4, 1, 6) (4,)	R^3 plane
$x = s - 2t$ $y = 3 - t$ $z = 2 - 4s + 5t$	para.	(0, 3, 2) (1, 3, -2)	R^3 plane

$4 = 1 + 2s - t$
$1 = -2 + 0s + t$
$3 = t$ $s = 3$

$0 = s - 2t$
$3 = 3 - t$
$0 = t$, $s = 0$

$1 = s - 2t$
$3 = 3 - t$
$0 = t$
$1 = s$.

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The vector equation of a plane in R_3

$<x, y, z> = <x_0, y_0, z_0> + s<a, b, c> + t<d, e, f>$

$<x, y, z>$ is the position vector of any point in the plane

$<x_0, y_0, z_0>$ is the position vector of a specific point in the plane

$<a, b, c>$, and $<d, e, f>$ are two <u>non-collinear</u> direction vectors of the plane

** symmetric form not possible

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Example 1: Find the vector equation of the plane that….

a) …contains the point P(1,2,3), and the line $\vec{r} = <2,-3,1> + t<1,0,2>$

P_1 P_2

$\vec{r} = <1,2,3> + s<1,0,2> + t<1,-5,-2>$

$\vec{P_1P_2} = <1,-5,-2>$

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Example 1: Find the vector equation of the plane that….

b) …contains the points A(0, 2, -2), B(1, -1, 5) and C(1, 1, 4)

$\vec{AB} = <1,-3,7> = \vec{d}_1$

$\vec{AC} = <1,-1,6> = \vec{d}_2$

$\vec{r} = <0,2,-2> + s<1,-3,7> + t<1,-1,6>$

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Example 1: Find the vector equation of the plane that….

c) …contains the point P(2, -3, 2) and is orthogonal to

$\vec{r} = <2,-3,1> + t<\underline{1,0,2}>$

$\langle a, b, c\rangle \cdot \langle 1, 0, 2\rangle = 0$

$a + 2c = 0$

Let $\Rightarrow a = 2$

$c = -1$

$\therefore \langle 2, 0, -1\rangle = \vec{d}_1$

$\langle e, f, g\rangle \cdot \langle 1, 0, 2\rangle = 0$

$e + 2g = 0$

Let $e = 6$, $g = -3$ this can be anyth but "0"

$\vec{d}_2 = \langle 6, 1, -3\rangle$

$\therefore \vec{r} = \langle 2, -3, 2\rangle + s\langle 2, 0, -1\rangle + t\langle 6, 1, -3\rangle$

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Example 2: Describe the following. (Line or plane…be careful)

$\vec{r} = <3, 2, 0> + s\underbrace{<3, -1, 1>}_{\vec{d}_1} + t\underbrace{<-6, 2, -2>}_{\vec{d}_2}$

$-2\vec{d}_1 = \vec{d}_2$

$\therefore$ They are collinear and not planar

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