Lesson 4: Scalar Equations of Planes in 3-Space

Warm Up: Determine three points that satisfy the equation

$2x - 4y + z = 10$ $A(1,1,12)$ $B(3,2,12)$
$C(3,5,24)$

a) How do we know this equation does not represent a line?

$x, y, z \Rightarrow R^3$

b) What figure does this equation represent?

plane.

May 16-2:37 PM

Example 1:

a) Use your 3 (non-collinear) points on the plane 2x - 4y + z = 10 to determine a vector equation of the plane.

b) Determine $\mathbf{d}_1 \times \mathbf{d}_2$. What do you notice?

c) Compare the results with neighbours'.

$\vec{r} = \langle 1, 1, 12 \rangle + s\langle 2, 1, 0 \rangle + t\langle 2, 4, 12 \rangle$

$\vec{AB} = \langle \quad \rangle = \vec{d_1}$

$\vec{AC} = \langle \quad \rangle = \vec{d_2}$

$\vec{d_1} \times \vec{d_2} = \langle 12, -24, 6 \rangle$ *

parallel normals

$\langle 25, -50, 12.5 \rangle$

$\langle -2, 4, -1 \rangle$ *

$\langle -6, 12, -3 \rangle$ *

May 24-2:18 PM

The **Scalar Equation** of the plane

The Scalar Equation of a plane is Ax + By + Cz + D = 0 where <A, B, C> is a vector normal to the plane.

Parallel planes have parallel normals.

Perpendicular planes have perpendicular normals.

May 16-2:39 PM

Example 2: Determine the Scalar equation of the plane

$$\begin{cases} x = -3s + t \\ y = 6s + 5 \\ z = 4 - 6s - 2t \end{cases}$$

$\vec{d}_1 = \langle -3, 6, -6 \rangle$

$\vec{d}_2 = \langle 1, 0, -2 \rangle$

$P = (0, 5, 4)$

$\vec{N} = \vec{d}_1 \times \vec{d}_2$

$= \langle -12, -12, -6 \rangle$
A B C

$Ax + By + Cz + D = 0$

$-12(0) + (-12)(5) + (-6)(4) + D = 0$

$D = 84$

$\therefore -12x - 12y - 6z + 84 = 0$

$-2x - 2y - z + 14 = 0$

May 16-2:40 PM

Example 3: Determine the scalar equation of the plane that contains the point (0, 2, 1) and is normal to <2, -1, 1>.

x y z   A B C

$2(0) + (-1)(2) + (1)(1) + D = 0$

$D = 1$

$\therefore 2x - y + z + 1 = 0$

May 16-2:40 PM

Example 4: Determine the angle between the planes

$2x - y + z = 5$

and $x + 3z = 2$

$\vec{n}_1 = \langle 2, -1, 1 \rangle$

$\vec{n}_2 = \langle 1, 0, 3 \rangle$

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$5 = \sqrt{6}\sqrt{10}\cos\theta$

$49.8^\circ \doteq \theta$

$\vec{n}_1 \cdot \vec{n}_2 = 5$

$|\vec{n}_1| = \sqrt{6}$

$|\vec{n}_2| = \sqrt{10}$

May 16-2:41 PM

May 30-12:09 PM