Lesson 6: Systems of Equations

Pre-Skill (Coplanar Vectors)

Vectors are coplanar if they lie in the same plane.

- Any two vectors are coplanar (& they can define a plane if they're not collinear)
- A 3rd vector is coplanar if it can be written as a combination of the other two (i.e. $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$)

The test for coplanarity:

Three vectors are coplanar if and only if $\boxed{\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0}$

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A **System of Equations** is a set of more than one equation.

A **Solution** to a system of equations is a set of points that satisfy all equations simultaneously.

Linear systems can have **one, none,** or **an infinite** amount of solutions.

Systems that have a solution are called **consistent.** Systems with no solutions are called **inconsistent**

Equivalent Systems are systems whose solutions are identical, a solution to one system is a solution to the other.

Equivalent systems can be produced by

a) Multiplying an equation by a non-zero constant
b) Interchanging two equations
c) Replacing an equation with the sum of itself and the scalar multiple of another.

Example 1: Solve

a) $2x - 5y = 2$ ×3
$6x - 8y = 13$

$$\begin{array}{r} 6x - 15y = 6 \\ -\quad 6x - 8y = 13 \\ \hline -7y = -7 \\ y = 1 \end{array}$$

$2x - 5(1) = 2$
$x = 3.5$

∴ Solution is $(3.5, 1)$

b) $2x + y + 6z = 7$ ×4
$3x + 4y + 3z = -8$
$x - 2y - 4z = 9$ ×2

$$\begin{array}{r} 8x + 4y + 24z = 28 \\ -\quad 3x + 4y + 3z = -8 \\ \hline 5x \quad + 21z = 36 \end{array}$$

$$\begin{array}{r} 3x + 4y + 3z = -8 \\ +\quad 2x - 4y - 8z = 18 \\ \hline 5x \quad - 5z = 10 \end{array}$$

$$\begin{array}{r} 5x + 21z = 36 \\ -\quad 5x - 5z = 10 \\ \hline 26z = 26 \\ z = 1 \end{array}$$

Find x:
$5x + 21(1) = 36$
$x = 3$

Find y:
$2(3) + y + 6(1) = 7$
$y = -5$

∴ Solution is
$(3, -5, 1)$

Example 2: Determine conditions on a and b, so that the system has…

$$2x + 3y = a$$
$$4x - by = 14$$

i) *no solution*
ii) *an infinite amount of solutions*
iii) *only one solution*

i) parallel lines
→ same slope

$2x + 3y = a$
$4x - by = 14$

$b = -6$ $a \neq 7$

ii) ∞ solutions
→ coincident lines
→ identical/scalar multiples

$a = 7$, $b = -6$

iii) 1 solution {diff. slopes}

$b \neq -6$, $a \in \mathbb{R}$

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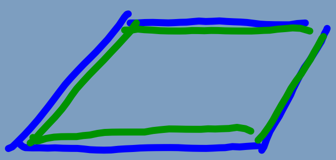
Intersecting TWO Planes

With 2 planes what are the possibilities for an intersection?
(Hint: 3 cases)

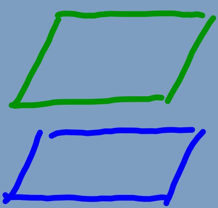


2 coincident planes

-> consistent

-> solution is the whole plane

-> Solve by inspection

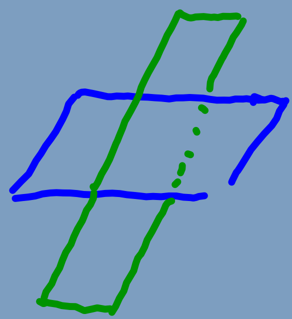


2 parallel planes

-> no solution

-> inconsistent

-> solve by inspection



2 Non-parallel planes

-> consistent

-> Solution is a line

-> classify by inspection

-> solve by finding the linear equation

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Example 3: Intersect the following pairs of planes.

a) $\pi_1 : 2x - y + z \cancel{-1} = \cancel{0}\ 1$

$\pi_2 : x + y + z = 6$

$3x + 2z = 7$

* NOT PARALLEL

↳ intersect in a line

$\boxed{\begin{aligned} x &= t \\ z &= 3.5 - 1.5t \\ y &= 2.5 + 0.5t \end{aligned}}$

Solution is a line in parametric form

@ $x = t$, $z = ?$

$3(t) + 2z = 7$

$z = 3.5 - 1.5t$

@ $x = t$, $z = 3.5 - 1.5t$

$y = ?$

$t + y + 3.5 - 1.5t = 6$

$y = 2.5 + 0.5t$

$$\pi_1 : 2x - y + z \cancel{-1} = \cancel{0}$$

b) $$\pi_2 : -6x + 3y - 3z = -3$$

planes are parallel + coincident

$\therefore$ the solution is

$2x - y + z = 1$

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Example 4: Give the equations of two planes whose intersection would satisfy the third case.

parallel but not coincident

$\pi_1 : 6x - 3y + 4z = 7$

$6x - 3y + 4z = 8$

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